VALUE AT RISK OF A LEHMAN BOND

Flavio Angelini
Dipartimento di Economia, University of Perugia, Italy
flavio.angelini@unipg.it

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Abstract

The main purpose of this paper is to show how a plain vanilla floating rate note (FRN) could have very high market risk, measured by Value at Risk, in presence of serious concerns about creditworthiness of the issuer. The issuer taken under consideration is Lehman Brothers and the analysis focuses expecially on the period before its failure, which was indeed a very critical period for the US bank system. The FRN considered was part of a list of bonds declared low risk/return by a consortium of italian banks called Patti Chiari. One of the requirement for a bond to be in the list was to have a low Value at Risk, not above a certain threshold, that was set at 0.3125% for the one-day 99% VaR. Italians investors, that lost their capitals after Lehman default, have been in dispute with their banks and with the Patti Chiari consortium because they argue that the VaR of their bonds had exceeded, from some months, the maximum threshold and therefore that those bonds would have to come out of the list. A simple and direct backtesting unequivocally shows that those cases have a foundation, in particular for the bond XS0189741001 which this document deals with. That is, that any measure of one-day 99% VaR that falls within the maximum limit of 0.3125% can be declared wrong with a margin of error close to zero.

Keywords: Value-at-risk, lehman brothers bond, historical simulation, ewma, backtesting.

1. INTRODUCTION

One fine day Scilla, a person who wants to be very cautious, invests the savings of a lifetime on a bond. Scilla feels quit safe, because the bond issued by the Lehman Brothers bank is part of a list of bonds declared low risk/return by Patti Chiari[1]. Patti Chiari was a consortium of italian banks founded on 2003, born with declared information and transparency objectives (Elenco delle obbligazioni a basso rischio-rendimento - Guida pratica, List of low-risk/yield bonds - Practical Guide). The next day she turns on the radio and discovers that the bank has failed. Her life, from that precise moment, changes.

Two things are meant with “risk"”. First, the risk of insolvency. This is managed through the issuer's rating, inserting in the list only high-rated issuers (Lehman had rating A). Secondly, the market risk, or the risk that changes in the price of the security can lead to losses on the investment. This is controlled through the Value-at-Risk, by including only bonds with low VaR, not above a certain threshold. This threshold was set at 0.3125% for the one-day VaR for a confidence level of 99%.

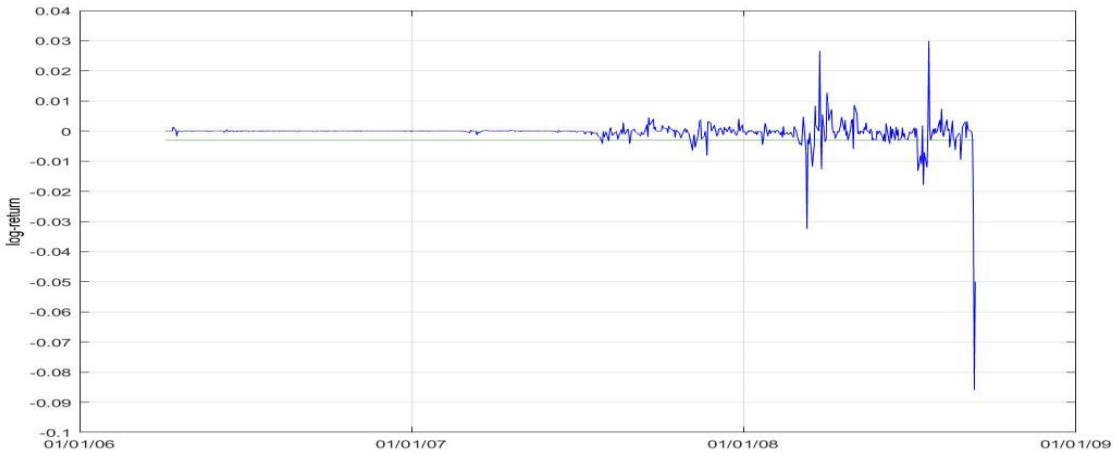
Along with Scilla there are several thousand small and medium-sized investors, for a total notional amount of several billions of euros, both simple buyers of Lehman bonds, and underwriters of insurance policies in which Lehman bonds were included as a guarantee of capital.

Many of the investors have been in dispute with their banks and with the Patti Chiari consortium which was responsible for drafting the list. There were cases, for example, at the courts of Padua, Milan, Imperia, some of which are still open. They argue that the VaR of their bonds had passed, abundantly and for several months, the maximum requirement threshold and therefore that those bonds would have to come out of the list.

A simple and direct backtesting, summarily represented in Figure 1, unequivocally shows that those cases have a foundation, in particular for the bond with ISIN code XS0189741001 which this document deals with. That is, that any measure of one-day 99% VaR for XS0189741001 that falls within the maximum limit of 0.3125% can be declared wrong with a margin of error close to zero.

[1] Literal transalation is ``clear agreements", and this comes from an italian saying: clear agreements and long friendship.

Figure 1
Daily logarithmic returns of the bond XS0189741001 from 6/4/2006 to 12/9/2008 and one-day VaR PattiChiari maximum limit (0.3125%) (Source price of the bond: Bloomberg)



This is true not only in retrospect, that is the day of bankruptcy, but it is also true from several months before, certainly starting from March 2008. For example, on 1/4/2008, looking at the previous 250 days, there are 19 violations of VaR, that is 7.6% of the time, against the hypothetical 1%. The probability that out of 250 observations there will be 19 violations of a proper 99% VaR is about 2 over10^{11}. Put another way, if backtesting were done each year, 19 violations in one year of a correct VaR would have been twice every 100 billion years. The 1/4/2008 was one of the days of revision of the Patti Chiari list, in which some bonds came out of the list, but not the Lehman ones.

The results of the backtesting procedure are illustrated in detail in Section 3, which is therefore the central paragraph of this note. In Section 4, VaR estimates of the bond are obtained by implementing standard methods, which take into account the past variability of the bond's market quotations. This section serves as confirmation, although superfluous, that the VaR had effectively exceeded the required threshold several months before the bankruptcy. But it also shows that the VaR of a plain vanilla fixed income security such a floating rate note can be significant and cannot be neglected, in presence of seriuos issues about creditworthiness of the issuer. All the results were obtained through MATLAB.

The Lehman default in Italy differs from the previous cases, Argentina, Parmalat, Cirio, precisely because some Lehman bonds, including the one analyzed in this note, were included in the Patti Chiari list. The goal of the list was that of selecting bonds on the basis of objective criteria that guaranteed their low risk, which criteria had to be continuously monitored. The investor would have been warned if any parameter had changed the measure of the riskiness of her investment or if her bond had left the list. These investors did not look for high-yield bonds, as it had been for the other cases mentioned.

Apart from the dramatic social and economic repercussions of the Lehman case, the problem is still current for small and medium-sized investors. Government and corporate bonds,

as they are typically simple and easily understandable, represent possible and attractive forms of investment, and are often offered by banks and financial intermediaries. As a matter of fact, the price fluctuations of these securities could be quite strong and are certainly not influenced mainly by the interest rates, but by factors related to the credit reliability of the particular issuer. This was for instance the case of bonds issued by many of the major US banks during the period this paper examines, in a similar way as the Lehman one showed here. The market risk of such securities could then be quite relevant and should be disclosed to the client in a transparent manner.

2. VAR AND PATTI CHIARI REQUIREMENT

A financial portfolio is subject to market risk, the risk of a decrease in the market value of the position. In this work we consider the market risk of a bond bought as an investement therefore the risk is that of a decrease in its market price. The main risk factors of corporate bonds are interest rate risk, which is due to the fluctuation of risk-free interest rates, and credit risk, that incorporates the risk of the default; or, in a mark to market approach, the creditworthiness of the issuer changes.

One of the most used indices for measuring market risk is Value at Risk. In particular, VaR with a confidence level of 99% is the object of the analysis. Put simply, the 99% VaR, is the relative loss which will be exceeded only 1% of the time (other times one refers to VaR as a loss in terms of capital). More precisely, given a time horizon of T days, ΔV is defined as the change in the value of the security (in general of the portfolio) in T days. Given a confidence level p, in our case p = 99% , the VaR at the confidence level p is defined as

$$Prob\left(\frac{\Delta V}{V} \leqslant -VaR\right) = 1 - p$$

where V is the value of the position at the time of evaluation and the minus sign merely serves to have a positive number as a VaR. The requirement to be in the Patti Chiari list was set to have a one-day 99% VaR lower than 0.3215%.

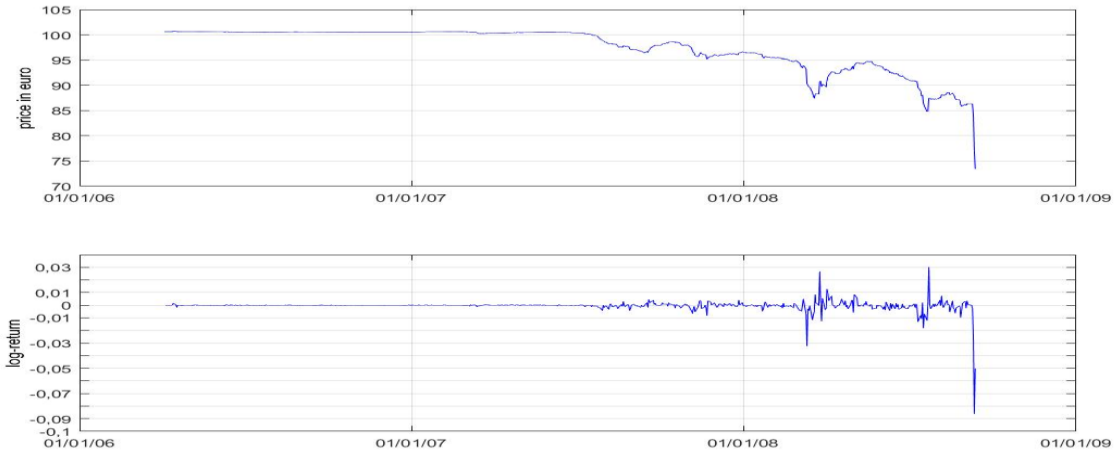
The crucial point of the VaR estimate obviously lies in the estimate of the ΔV distribution. Although there is a large scientific literature for estimating this distribution and therefore for the estimation of VaR, there is no doubt that an operating standard is represented by the models and methodologies proposed by RiskMetrics. But there is no “the method” for estimating VaR. In the RiskMetrics documents reference is made to different models and methods. In Mina and Xiao (2001), Chapter 7, it is also suggested to report the results of different methods for the risk analysis of the portfolio or security under examination. In fact, the comparison of results from different methods is useful to study the effect of distributional assumption, and estimate the potential magnitude of the error incurred by the use of the model. For these reasons, in addition, any model should be tested, which is what it is dealt with in Section 3.

The bond taken as example in this paper is a plain vanilla floating rate note denominated in euro, with ISIN code XS0189741001. It was issued by Lehman Brothers on 5/4/2004 with maturity on 5/4/2011. The bond is indexed to the three-month Euribor plus a spread of 0.35%, with coupons on a quarterly basis. The daily prices of the bond, represented in the chart at the top of Figure 2, refer to the period from 5/4/2006 to 12/9/2008 (day of market opening preceding the declaration of bankruptcy). In the bottom chart of Figure 2 the daily logarithmic

returns are shown. It is worth to remember that theoretically, a floating rate note like that, if it were default free, would have very low market risk. The fluctuation of the price would be that of the first period of the sample, in 2006 and in the first half of 2007.

Figure 2

Time series of prices from 5/4/2006 to 12/9/2008 (top) and log-returns (bottom) of bond XS0189741001 (Source Bloomberg)



However, it is evident at glance that the series of logarithmic returns of the bond is not stationary. We can distinguish at least two periods, separated around the middle of 2007. But we also see a further change of regime in the first months of 2008, between February and March. It is also clear that the series has fat tails. For example, in about two and a half years, for 637 observations, considering the average μ and the standard deviation σ estimated on the entire sample, there are 3 values less than μ-5 σ , around 0.5% of the sample, which can then be considered extreme events.

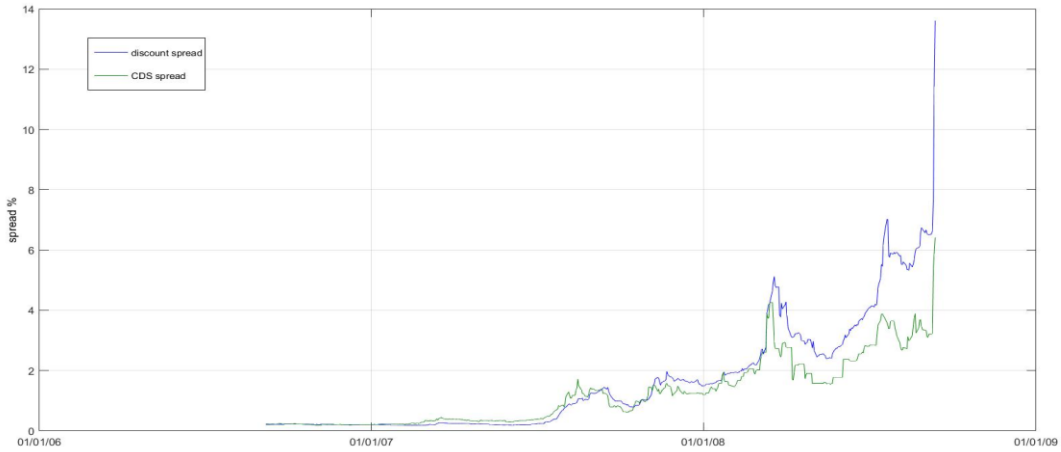
Given the non stationary character of the series, the standard statistics have been calculated over the last 250 days and then moved day by day with a moving window. Over time, volatility on a yearly basis has risen from under 1% in the first half of 2007 to reach 2% in late 2007, up to around 6% in the first part of 2008 and reach and exceed 7 % in the summer of 2008. The asymmetry is always negative and also reaches the values -5 and -6. Kurtosis is very high, in the central part of 2007 around 40, for a period of 2008 it reaches almost 100. On the whole sample we have a kurtosis of 153. The kurtosis provides a measure of the fatness of the tails of the distribution of returns, and the above values confirm the presence of extreme events.

The other Lehman bonds that were included in the list as of September 13, 2008 are: XS0179304869, XS0193035358, XS0205185456, XS0252834576, XS0252835110, XS0128857413, XS0272543900, XS0138439616, XS0300055547, XS0224346592, XS0257022714, XS0282937985. I have the time series of quotes of all of them and one could therefore carry out the same analysis as for XS0189741001 and show that they also do not have low risk profiles, at least in the last months. All the data were provided by Bloomberg.

Now we shall briefly analyze whether the interest rate risk is sufficient to explain the variability of the bond market prices, hence to correctly measure the market risk. To do so we will compute the discount spread, or zero-discount margin, with respect to the inter-bank interest rate curve, calibrated on bond prices (see O'Kane and Sen (2005)). This is represented in Figure 3: in the first part of the sample the spread is low, around 20 bps, while in the second

part of 2007 it begins to raise, oscillating between 100 and 200 bps. Since February 2008 it has steadily exceeded 200 bps, up to even exceeding 600 bps. This measures how the bond's prices are mainly explained by factors related to the issuer and not by interest rates. We then consider the 5Y Lehman Brothers Credit Default Swap (CDS) spread, as another measure of creditworthiness. As explained in Hull (2018), Par. 24.1, the excess of a bond yield over the risk-free rate should approximately equal the same maturity CDS spread. It makes then sense to compare CDS spreads with discount spreads. This is shown in Figure 3 and it is quite remarkable the very similar behaviour of the two quantities, both indicating a deterioration of the creditworthiness of Lehman Brothers in the second half of 2007 and in 2008, Similar behaviours could also be observed in the case of CDS spreads of the major US banks.

Figure 3
Calibrated Discount Spread and 5Y CDS spread



3. BACKTESTING

Given the wide range of methodologies and models for estimating VaR, the results of the procedures must be subjected to verification tests. The backtesting is a statistical test that is used to verify the VaR estimation model, and falls into the hypothesis testing category. It is recommended to banks by the Basel Accords, drafted by the Basel Committee, as a control of their VaR estimation systems for the definition of capital requirements (see the Basel Committee document (1996)).

More importantly, in the context of this note, backtesting was part of the second phase of the process of composing the Patti Chiari list: “Having identified the subclasses based on the characteristics of the bond [...], for each subclass a “theoretical” bond is abstracted and the VaR is determined. The Processor then associates the theoretical VaRs with the individual bonds. The behavior of the individual bond is therefore verified with respect to the “theoretical” one (backtesting procedure) " (page 19 of the Guide to the banking operator, Guida all'operatore bancario, translated from italian). It turned out that backtesting was never implemented.

The test in general wants to verify the hypothesis that the VaR estimate at a given confidence level carried out by the model is correct. It should be emphasized that the statistical tests on VaR, as a percentile, are rather delicate, and sometimes it is difficult to interpret the results. A discussion of these limitations and problems can be found in the scientific literature,

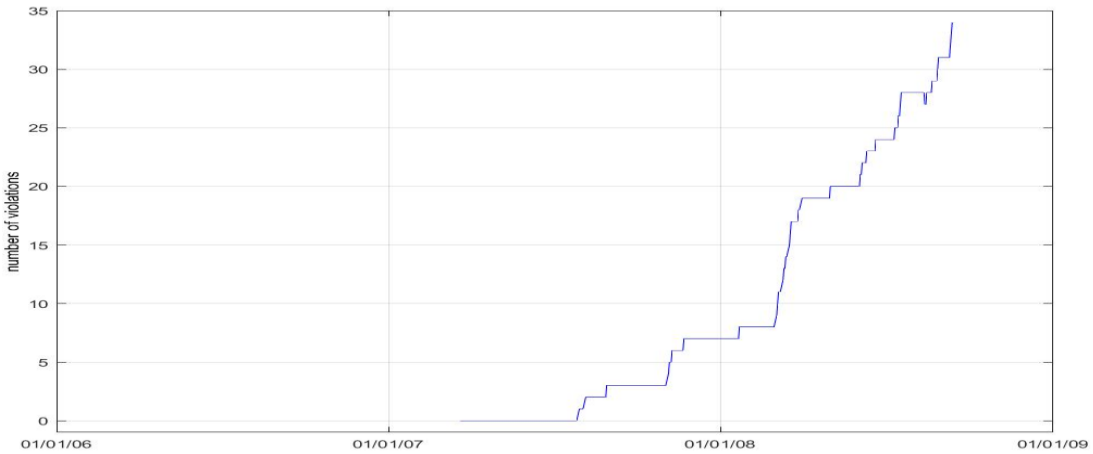
but is also contained in the Basel Committee document (1996). There are two risks in the answers, accepting the hypothesis when the estimate is not accurate, or reject the hypothesis when the estimate is accurate. The latter is called the type I error and is the one of interest in this discussion. In the case in question the results are so strong that they are hardly questionable.

With reference to the requirements of the Patti Chiari list, the hypothesis to be tested is: the one-day 99% VaR of the bond has never exceeded 0.3125 %. Of course the test should be performed on the actual VaR estimates performed by the PattiChiari consortium, which unfortunately are not available. However, the rejection of the maximum VaR threshold brings a greater reason to reject the actual estimate and with greater probability of not incurring the type I error.

For backtesting, the Basel Committee recommends the use of one year of past daily observations, or 250 observations. In short, a given day, the past 250 days are considered and the VaR estimates are produced with the model used for all these days. In the case in question we have a value set at the maximum threshold of 0.3125 %. The past performance of the bond returns is observed and the number of violations is counted, when the return on a given day exceeds the VaR estimate made the previous day. The test is based on the hypothesis that violations are independent over time. The Basel Committee (1996) defines three zones, the green zone (0-4 violations), the yellow zone (5-9 violations) and the red zone (more than 10 violations). For the red zone, the Basel Committee believes that outcomes in the red zone should generally lead to an automatic presumption that a problem exists with a bank's model. It is indeed extremely unlikely that an accurate model would independently generate ten or more exceptions from a sample of 250 trading outcomes.

The procedure is illustrated in Figure 1 on the whole sample under examination, where the time seires of logarithmic returns of the bond are shown. The VaR level of -0.3125 % is highlighted with a horizontal line. The violations are all exceedances by the returns of the horizontal line. The test was carried out every day starting 250 days after the first date of the sample of returns, which is 6/4/2006, then from 3/21/2007 to 12/9/2008. The results are shown in Figure 4. In the first period (at least until mid 2007) there are no violations. The violations begin in the second half of 2007, initially fewer than 5, maintaining the estimates in the so-called green zone. Between 2007 and 2008, the test begins to produce violations in numbers greater than or equal to 5, therefore entering the yellow zone. Starting from March 3, 2008, you enter the red zone and you do not get out any more, reaching the 30 violations a few days before the bankruptcy. For example, on 1/4/2008, the revision date of the Patti Chiari list, there are 19 violations. The likelihood of this happening is about over 10^{11}. If backtesting was to be done each year, 19 exceedances in a year of a correct VaR would happen twice per 100 billion years.

Figure 4
Backtesting: number of violations of the maximum threshold 0.3125% of one-day 99% VaR based on 250 past observations



In conclusion, the backtesting, carried out on any day between 3/3/2008 and 12/9/2008, rejects the hypothesis that the daily 99% VaR of the bond is 0.3125% or lower. The likelihood of rejecting a correct estimate is remote. In particular, if on 1/4/2008, the day on which the list was revised, someone had backtested an estimate of VaR that fell within the limit requirement, this estimate would have been rejected with a confidence level of 99.999999998%. In other words, the maximum VaR requirement threshold, and therefore any VaR estimates below this, largely end up in the so-called red zone. Therefore, any 99% daily VaR measure for the bond that falls within the 0.3125% maximum limit may be declared incorrect with a very ample confidence margin.

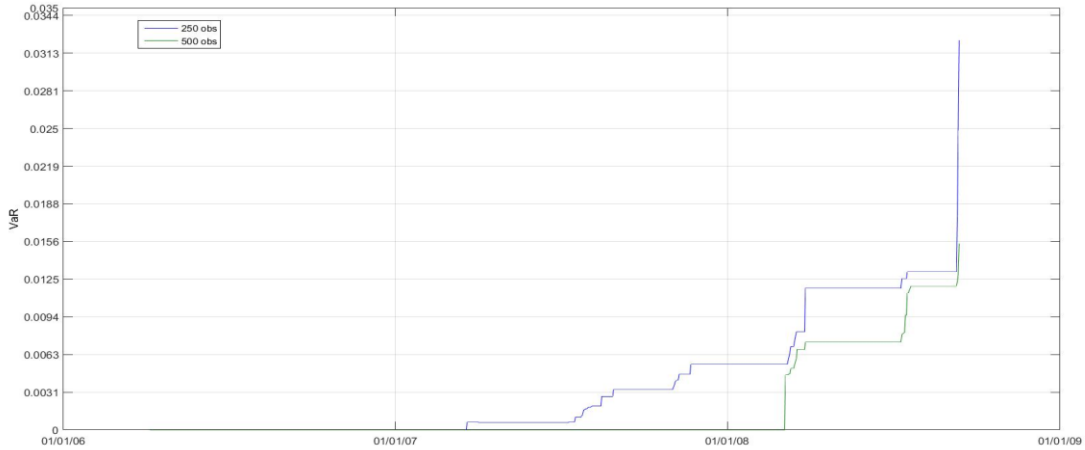
4. STANDARD METHODS FOR VAR ESTIMATES

To get a first sensible estimate of one-day 99% VaR of the bond, standard methods can be applied among those most used by banks for their portfolios, namely RiskMetrics model with volatility estimation through EWMA and historical simulation. Such models are backtested in the papers of the MSCI group. For instance those by Finger and Abbasi (2012) or Vajda et al. (2019). Vlaar (2000) uses those models to investigate the VaR of Dutch bond portfolios. We will also adopt the approach described by Mina and Xiao (2001) in Ch. 1, and model directly the time series of prices of the bond. Instead of utilizing a model based on an underlying yield curve. The methods used here are also in line with the definition of the Patti Chiari banking operator Guide so that, translating from italian, ``the market risk of a listed financial instrument can be estimated by measuring an indicator of the variability (volatility) shown in the past by the bond market prices, an indicator that provides a measure of the expected variability of the value of that security in the future. One of the most notable is the VaR index ..."

The results of the one-day VaR estimate using historical simulation of the bond are shown in Figure 5, using 250 and 500 past observations. The estimates then start after

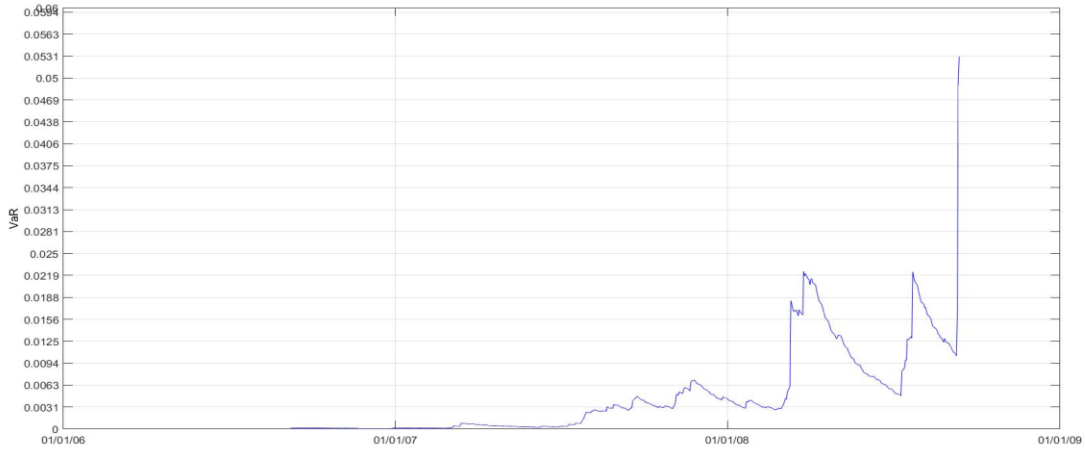
respectively 250 and 500 days from the beginning of the sample.

Figure 5
Estimates of the one-day 99 % VaR of the bond using the historical simulation method, using 250 and 500 past observations. Estimates therefore start at 250 or 500 days after the beginning of the sample



The results of the one-day VaR estimates using the RiskMetrics model are shown in Figure 6. The EWMA parameter was set at $\lambda = 0.94$, in line with the RiskMetrics indications.

Figure 6
Estimates of the one-day 99% VaR of the bond via the RM-EWMA model. Estimates start at 112 days after the beginning of the sample



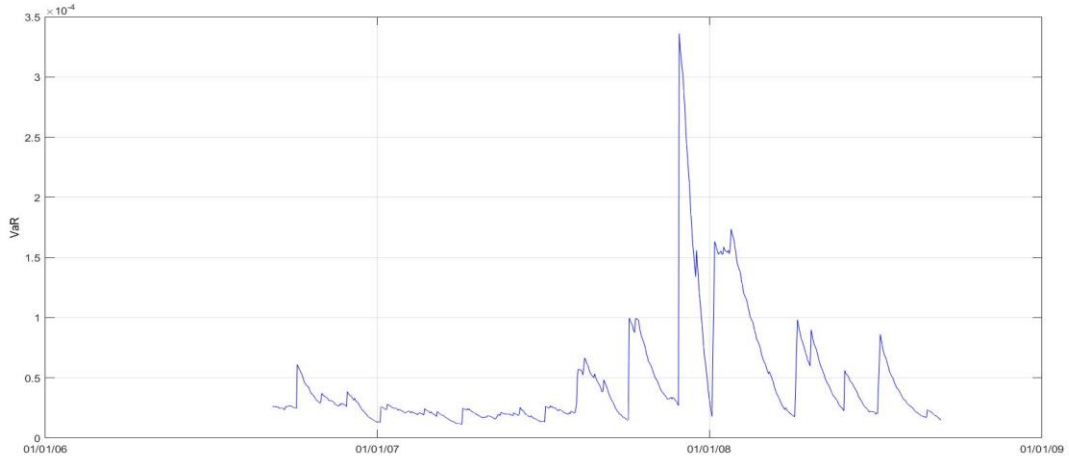
Estimates through historical simulation, although not exceeding 1.3% daily (apart from the last exceptional day), place the VaR steadily above the threshold of 0.3125% from around the end of 2007, and largely above threshold since March 2008. The RM-EWMA model also produces estimates above the threshold in the same periods, but with a more variable trend, sometimes exceeding values of 2%.

Comparing the results obtained with the two methods, we can see that those obtained with the historical simulation method are less reactive to regime variations, since they take uniformly account of all the past daily changes, even those preceding the half of the 2007 (particularly using 500 past observations) in which the price of the bond had another pattern.

Alternatively, one could think of implementing a method based on the identification of risk factors and on a pricing formula that depends on these factors. If one decides to use only the interest rate curve as a risk factor, implementing a linear approximation method and using EWMA variance-covariance matrix estimates for changes in interest rates, the corresponding VaR estimates that would come out are represented in Figure 7. As can be seen, the maximum values reached by the estimates are of the order of $3.5 \cdot 10^{-4}$, that is at least an order of magnitude lower than the previous estimates. This is in line with the fact that the bond, if the issuer's factors are not considered, such as its creditworthiness, is similar to a security with a maturity lower than a quarter. This method does not take into account the variability of the past bond prices and therefore leads to seriously underestimated values.

To take into account the factors linked to the issuer, one can, in line with Mina and Xiao (2001), determine a calibrated spread, that would take the meaning of credit spread. The pricing formula is described for example in O'Kane and Sen (2005) and the spread (called zero-discount margin) should be calculated in order to reproduce the bond's market prices, given the interest rate curve. Such a spread was that shown in the Figure 3. By applying the linear approximation method and EWMA estimates of the variance-covariance matrix of changes in risk factors, interest rates and spreads, VAR estimates can be made. It is not surprising that these estimates are consistent with those obtained with the EWMA method applied directly to the bonds' prices, therefore they will not be shown here.

Figure 7
Estimates of the one-day 99% VaR of the bond using the RM parametric method with interest rate risk only, without inserting the credit spread



5. CONCLUSION

The bonds issued by Lehman Brothers, in particular the one with ISIN code XS0189741001 analyzed in detail in this note, could not be considered low risk, at least in the

last months before bankruptcy. In particular, in the last few months, they did not meet the VaR requirement to be included in the list drawn up and controlled by the Patti Chiari consortium. The problem, apart from the dramatic social and economic consequences of the Lehman case, is current and concerns savers, given that government or corporate bonds, typically simple and easily understandable, represent possible and attractive forms of investment, and are often proposed from intermediaries to their retail customers. Their market risk instead could be quite strong and should be disclosed to the client in a transparent manner. Future research should then again address the problem of measuring the risk of government and corporate bonds. To this aim, one should also implement more appropriate models than the standard ones used in the present paper in order to incorporate creditwhortiness indexes coming from the market, like for instance CDS spreads.

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