Lower and Upper Bounds of the Ultimate Ruin Probability in a Discrete Time Risk Model with Proportional Reinsurance and Investment

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Abstract

The lower and upper bounds of the ultimate ruin probability in a discrete time risk model with proportional reinsurance and investment are determined under the assumption that the reinsurance retention level and the amount of investment in a particular stock during each time period can remain constant by employing the integral operator L. The lower bound is obtained from the finite time ruin probability that converges to the ultimate ruin probability with increasing time while the upper bound is iteratively determined by using Luesamai and Chongcharoen's upper bound as the starting point. Besides, the ultimate ruin probability as a fixed point of L is illustrated.

Keywords: Reinsurance and Investment, Bound of Ruin Probability, Discrete Time Risk Model

1 Introduction

A well-known classical discrete time risk/surplus model for insurance assessment is specified as

$$U_n = U_{n-1} + c - Y_n, \quad n = 1,2,3, \ldots, \tag{1}$$

where U_n is the surplus at the end of time period n (from $n-1$ to n) with initial constant $U_0 = u \geq 0$; Y_n is the total claim amount during n, where Y_n are independent and identically distributed (i.i.d.) random variables with common distribution function $P(y) = Pr(Y_n \leq y)$, for $y \geq 0$; and c is the constant premium income per unit time (Cai & Dickson, 2004; Lin, Dongjin, & Yanru, 2015). Luesamai and Chongcharoen (2018) applied this model by adding proportional reinsurance as well as stock and bond investment factors; the output is defined as

$$U_n = U_{n-1}(1 + I_n) + \alpha_n W_n + c(b_n) - h(b_n, Y_n); \quad n = 1,2,3, \ldots, \tag{2}$$

where $b_n \in (0,1]$ is the retention level of proportional reinsurance in one period. Function $0 \leq h(b_n, Y_n) \leq Y_n$ specifies the fraction of the total claim amount Y_n paid by the insurer, with $G(y) = Pr[h(b_n, Y_n) \leq y]$, and it also depends on reinsurance retention level b_n at the beginning of the period. Hence, $Y_n - h(b_n, Y_n)$ is the part paid by the reinsurer. Reinsurance retention level $b_n = 1$ means that there is no reinsurance. In this model, only proportional reinsurance is considered, which means that $h(b_n, Y_n) = b_n Y_n$. Function $c(b_n)$, which is the premium retained by the insurer, is calculated as $c - c_{re}$. By the expected value principle, the constant premium income is calculated as $c = (1+\theta)E(Y_n)$, where $\theta > 0$ (the safety loading) is added by the insurer. The constant premium for the reinsurer is calculated as $c_{re} = (1+\delta)E[Y_n - h(b_n, Y_n)]$, where δ is the safety loading added by the reinsurer. Thus,

$$c(b_n) = [(1+\theta) - (1+\delta)(1-b_n)]E(Y_n). \tag{3}$$

Furthermore, in (2), I_n (the bond interest rate at time n) is assumed to follow a time-homogeneous Markov chain, where $I_n = i_k$, $k \in \{0,1,2,3, \ldots, d_n = d\}$ for all n and where I_0 is known; W_n (the gross return for the stock investment at time n) is assumed to be a sequence of i.i.d. nonnegative random variables with the distribution function $F(w) = Pr(W_n \leq w)$, where $w \geq 0$; and α_n is the amount of money that the insurer invests in stock at the beginning of the n^{th} period identified by using information from $\{I_j \text{and } W_j : j = 0,1,2, \ldots, n-1\}$.

Ruin probability is the probability that the insurer's surplus falls below zero at some time in the future (Dickson, 2005). Thus, the ruin probability for a finite time period is given by (Jasiulewicz & Kordecki, 2015; Luesamai & Chongcharoen, 2018)

$$\psi_n(u, i_s) = Pr\{U_k < 0 \text{ for some } 1 \leq k \leq n | U_0 = u, I_0 = i_s\} \tag{4}$$

$$= Pr\{\cup_{k=1}^{n}(U_k < 0) | U_0 = u, I_0 = i_s\},$$

while the ultimate ruin probability is given by

$$\psi(u,i_s) = Pr\{U_k < 0 \text{ for some } k \geq 1 | U_0 = u, I_0 = i_s\} \tag{5}$$

$$= Pr\{\cup_{k=1}^{\infty}(U_k < 0) | U_0 = u, I_0 = i_s\}.$$

Clearly, $\psi_1(u,i_s) \leq \psi_2(u,i_s) \leq \psi_3(u,i_s) \leq \cdots$ and

$$\lim_{n\to\infty} \psi_n(u,i_s) = \psi(u,i_s). \tag{6}$$

According to the previously mentioned assumptions and the Lundberg coefficient (adjustment coefficient), r_0 of the classical risk model exists. The r_0 is the smallest positive value of real variable r satisfying the equation

$$M_{Y_n-c}(r) = E\left[e^{r(Y_n-c)}\right] = 1. \tag{7}$$

Luesamai and Chongcharoen (2018) showed that if the retention level of reinsurance (b_n) and the amount of investment in stock (α_n) in each time period for the risk model in (2) are set as constant values, i.e.

$$U_n = U_{n-1}(1+I_n) + \alpha W_n + c(b) - bY_n; \quad n = 1,2,3,\ldots, \tag{8}$$

the upper bound of the ruin probability in (4) and (5) will become

$$\psi_{n+1}(u,i_s) \leq \psi(u,i_s) \leq \beta_0 E\left(e^{-r_0[u(1+I_1)+\alpha W_1]} | I_0 = i_s\right), \tag{9}$$

where

$$\beta_0^{-1} = \inf_{m\geq 0} \frac{\int_m^\infty e^{r_0 y} dG(y)}{e^{r_0 m}\bar{G}(m)}. \tag{10}$$

In this study, a new lower bound and an upper bound based on the assumptions of Luesamai and Chongcharoen (2018) are provided by applying the methodology of iteration and the integral operator L (Gajek, 2005; Rudź, 2015) which is defined as follows.

Let $\mathcal{R}$ be the set of all non-increasing functions defined on $[0,\infty)$ and taking values from $[0,1]$. Function $L: \mathcal{R} \to \mathcal{R}$ is said to be an integral operator generated by the risk model $\{U_n\}$ defined in (8) when

$$LR(u,i_s) = \sum_{t=0}^{d} p_{st} \int_0^\infty \int_0^\pi R[u(1+i_t) + \alpha w - z(y), i_t] dG(y) dF(w)$$

$$+ \sum_{t=0}^{d} p_{st} \int_0^\infty \bar{G}(\pi) dF(w), \tag{11}$$

where $z(h(b,Y_1)) = h(b,Y_1) - c(b)$ and $\pi = u(1+i_t) + \alpha w + c(b)$ are defined. These definitions are used throughout the paper. Obviously, L is monotone; i.e.,

$$LR(u,i_s) \leq LS(u,i_s), \tag{12}$$

for all $R(u,i_s) \leq S(u,i_s)$, where $u, i_s \geq 0$.

2 Iteratively determining the lower and upper bounds

In this section, the lower and upper bounds are derived by using the integral operator L adapted from the one defined by Gajek (2005). Under the assumptions that the retention level of reinsurance and the amount of investment in stock in each time period for the risk model in (2) are set as constant values (i.e., $b_n = b$ and $\alpha_n = \alpha$, for all $n = 1,2,3,\dots$), then the lower bound is provided by Theorem 1. Moreover, the upper bound for the ruin probability provided in Theorem 2 is iteratively obtained by using Luesamai and Chongcharoen's (2018) upper bound as the starting point.

Theorem 1: Let L be the integral operator defined in (11), and the ruin probability for a finite time, $\psi_n(u,i_s)$, be defined in (4) for the risk model in (8). Then

(i) for all $u, i_s \geq 0$, it holds that

$$\psi_n(u,i_s) = L^{n-1}\,\psi_1(u,i_s), \tag{13}$$

where $\psi_1(u,i_s) = \sum_{t=0}^{d} p_{st} \int_0^\infty \bar{G}(\pi) dF(w)$ and L^0 denotes the identity operator, and

(ii) the ultimate ruin probability, $\psi(u,i_s)$, is a fixed point of L; i.e.,

$$\psi(u,i_s) = L\psi(u,i_s). \tag{14}$$

Proof

(i) For $n = 1$, the result obviously holds because L^0 is the identity operator; i.e.,

$$\psi_1(u,i_s) = L^0\,\psi_1(u,i_s).$$

Assuming that (13) is true for some $n \in N$, then consider that when $n+1$, the result holds as

$$\psi_{n+1}(u,i_s) = \sum_{t=0}^{d} p_{st} \int_0^\infty \int_0^\pi \psi_n\,[u(1+i_t) + \alpha w - z(y), i_t] dG(y) dF(w)$$

$$+ \sum_{t=0}^{d} p_{st} \int_0^\infty \bar{G}(\pi) dF(w)$$

$$= L\psi_n(u,i_s)$$

$$= LL^{n-1}\,\psi_1(u,i_s)$$

$$= L^n\,\psi_1(u,i_s).$$

(ii)

$$\psi(u,i_s) = \lim_{n\to\infty} \psi_{n+1}(u,i_s)$$

$$= \lim_{n\to\infty} \sum_{t=0}^{d} p_{st} \int_0^\infty \int_0^\pi \psi_n\,[u(1+i_t)+\alpha w - z(y), i_t]dG(y)dF(w)$$

$$+ \sum_{t=0}^{d} p_{st} \int_0^\infty \bar{G}(\pi)dF(w)$$

$$= \sum_{t=0}^{d} p_{st} \int_0^\infty \int_0^\pi \psi\,[u(1+i_t)+\alpha w - z(y), i_t]dG(y)dF(w)$$

$$+ \sum_{t=0}^{d} p_{st} \int_0^\infty \bar{G}(\pi)dF(w) \text{ (by using Lebesgue's dominated convergence theorem)}$$

$$= L\psi(u,i_s).$$

Thus, $\psi(u,i_s) = L\psi(u,i_s)$, i.e. $\psi(u,i_s)$ is a fixed point of the operator L.

Theorem 2: Assume that r_0 in (7) exists. Let $R_0(u,i_s) = \beta_0 E\left(e^{-r_0[u(1+I_1)+\alpha W_1]} | I_0 = i_s\right)$ where β_0 is the inverse of (10). Then the sequence of iterations $R_n(u,i_s) = L^n\, R_0(u,i_s)$ is a non-increasing sequence of functions in $\mathcal{R}$ which converge monotonically from above to a fixed point of L.

Proof

For $n = 1$, $R_1(u,i_s)$ from $R_0(u,i_s) \in \mathcal{R}$ and L (its monotonicity will be use below) becomes

$$R_1(u,i_s) = LR_0(u,i_s)$$

$$= \sum_{t=0}^{d} p_{st} \int_0^\infty \int_0^\pi R_0[u(1+i_t)+\alpha w - z(y), i_t]dG(y)dF(w)$$

$$+ \sum_{t=0}^{d} p_{st} \int_0^\infty \bar{G}(\pi)dF(w)$$

$$= \sum_{t=0}^{d} p_{st} \int_0^\infty \int_0^\pi \beta_0 E\left(e^{-r_0[\{u(1+i_t)+\alpha w - z(y)\}(1+I_1)+\alpha W_1]} | I_0 = i_t\right)dG(y)dF(w)$$

$$+ \sum_{t=0}^{d} p_{st} \int_0^\infty \bar{G}(\pi)dF(w)$$

$$\leq \sum_{t=0}^{d} p_{st} \int_{0}^{\infty} \int_{0}^{\pi} \beta_0 e^{-r_0\{u(1+i_t)+\alpha w - z(y)\}} dG(y) dF(w)$$

$$+ \sum_{t=0}^{d} p_{st} \int_{0}^{\infty} \bar{G}(\pi) dF(w). \tag{15}$$

Consider $\bar{G}(m) = \left(\frac{\int_m^\infty e^{r_0 y} dG(y)}{e^{r_0 m} \bar{G}(m)} \right)^{-1} e^{-r_0 m} \int_{m}^{\infty} e^{r_0 y} dG(y)$

$$\leq \beta_0 e^{-r_0 m} \int_{m}^{\infty} e^{r_0 y} dG(y), \qquad \text{where} \quad \beta_0^{-1} = \inf_{m \geq 0} \frac{\int_m^\infty e^{r_0 y} dG(y)}{e^{r_0 m} \bar{G}(m)}. \tag{16}$$

By replacing (16) in (15), we obtain

$$R_1(u, i_s) \leq \sum_{t=0}^{d} p_{st} \int_{0}^{\infty} \int_{0}^{\pi} \beta_0 e^{-r_0\{u(1+i_t)+\alpha w - z(y)\}} dG(y) dF(w)$$

$$+ \sum_{t=0}^{d} p_{st} \int_{0}^{\infty} \beta_0 e^{-r_0[u(1+i_t)+\alpha w + c(b)]} \int_{\pi}^{\infty} e^{r_0 y} dG(y)\, dF(w)$$

$$= \sum_{t=0}^{d} p_{st} \int_{0}^{\infty} \beta_0 e^{-r_0\{u(1+i_t)+\alpha w + c(b)\}} \int_{0}^{\infty} e^{r_0 y} dG(y)\, dF(w)$$

$$= \sum_{t=0}^{d} p_{st} \int_{0}^{\infty} \beta_0 e^{-r_0\{u(1+i_t)+\alpha w + c(b)\}} E\left(e^{r_0 h(b, Y_1)}\right) dF(w)$$

$$= \beta_0 E\left(e^{-r_0[c(b) - h(b, Y_1)]}\right) E\left(e^{-r_0[u(1+I_1)+\alpha W_1]} | I_0 = i_s\right)$$

$$= \beta_0 E\left(e^{-r_0[u(1+I_1)+\alpha W_1]} | I_0 = i_s\right) \quad \text{(Diasparra and Romera, 2009, p.102)}$$

$$= R_0(u, i_s).$$

Assume that $R_n(u, i_s) = L^n R_0(u, i_s) \leq L^{n-1} R_0(u, i_s) = R_{n-1}(u, i_s)$.

Thus, for $n + 1$, the result holds as follows:

$$R_{n+1}(u, i_s) = L R_n(u, i_s)$$

$$\leq L L^{n-1} R_0(u, i_s)$$

$$= L^n R_0(u, i_s)$$

$$= R_n(u, i_s).$$

Hence, $\{R_n\}$ is a non-increasing sequence of functions in $\mathcal{R}$ that is bounded from below by 0. Consequently, the pointwise limit of $R_n(u, i_s)$ can be obtained as $R^*(u, i_s)$, such that

$$\begin{aligned}
R^*(u, i_s) &= \lim_{n\to\infty} R_{n+1}(u, i_s) \\
&= \lim_{n\to\infty} \sum_{t=0}^{d} p_{st} \int_0^{\infty}\int_0^{\pi} R_n[u(1+i_t) + \alpha w - z(y), i_t]dG(y)dF(w) \\
&\quad + \sum_{t=0}^{d} p_{st} \int_0^{\infty} \bar{G}(\pi)dF(w) \\
&= \sum_{t=0}^{d} p_{st} \int_0^{\infty}\int_0^{\pi} R^*[u(1+i_t) + \alpha w - z(y), i_t]dG(y)dF(w) \\
&\quad + \sum_{t=0}^{d} p_{st} \int_0^{\infty} \bar{G}(\pi)dF(w) \text{ (by using Lebesgue's dominated convergence theorem)} \\
&= LR^*(u, i_s).
\end{aligned}$$

Thus, $R^*(u, i_s) = LR^*(u, i_s)$, i.e. $R^*(u, i_s)$ is a fixed point of the operator L.

3 Numerical examples

Here, two numerical examples are presented to show the efficacy of applying the bounds provided in the previous section. In Example 1, the total claim amounts are assumed to be i.i.d. exponential. Application of iterative lower and upper bounds proposed in the previous section and the upper bound derived by Luesamai and Chongcharoen (2018) is illustrated for various amounts of initial surplus u. In Example 2, the total claim amounts are assumed to be i.i.d. normal. Application of iterative bounds derived in the previous section and the upper bound derived by Luesamai and Chongcharoen (2018) are presented for different values of b and α, where b is the constant value assigned to the retention level of reinsurance in each time period b_n and α is the constant value assigned to the amount of money invested in stock in each time period α_n.

Example 1: For all $n = 1,2,3, \ldots$, suppose that the total claim amount during period n is $Y_n \sim exp(1.25)$, the bond interest rate at time n is $I_n \in \{0.01, 0.015, 0,02\}$ with transition probability matrix

$$\begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.4 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.2 \end{bmatrix},$$

and $I_0 = 0.015$, and the gross stock return at time n is given by $W_n = e^{\left(\mu-\frac{\sigma^2}{2}\right)+\sigma B_n}$, where $B_n \sim N(0,1)$, $\mu = 0.9$, and $\sigma = 0.2$. The safety loading values given by the insurer (θ) and the reinsurer (δ) were assumed to be 10% and 11%, respectively. The policies for the reinsurance and the amount of investment in stock are assumed to be the same in each time period (i.e., $b_n = b = 0.7$ and $\alpha_n = \alpha = 0.005$, for all $n = 1,2,3,\ldots$).

The graphs of $R_1(u,i_s)$, $R_2(u,i_s)$, $R_3(u,i_s)$ and $\psi_1(u,i_s)$, $\psi_2(u,i_s)$, $\psi_3(u,i_s)$ are illustrated in Figure 1.

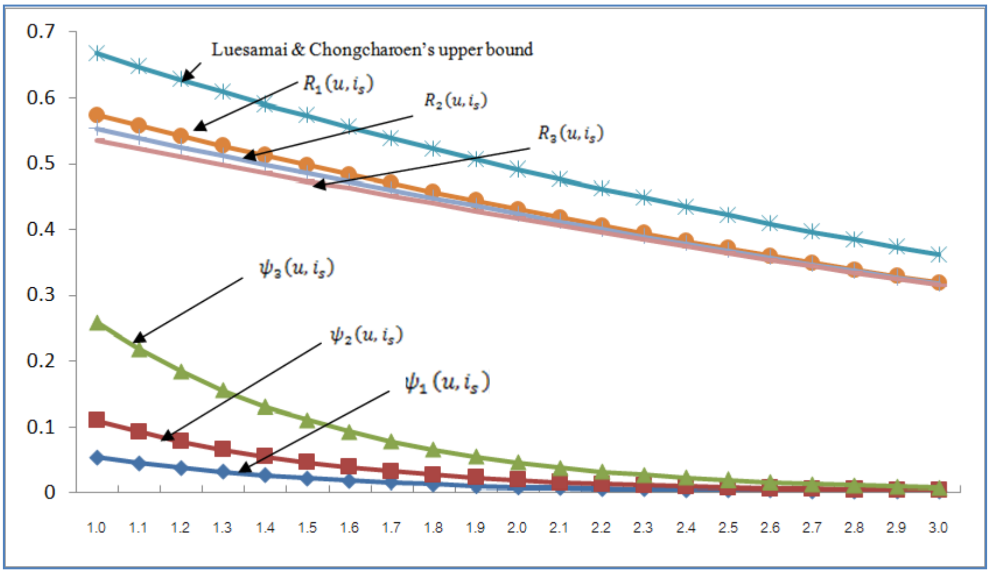

Figure 1: The first three iteratively determined upper $(R_n(u,i_s))$ and lower $(\psi_n(u,i_s))$ bounds of the ultimate ruin probability together with Luesamai and Chongcharoen's (2018) upper bound for varying initial surplus u.

It can be seen that the proposed upper bound sequence $(R_n(u,i_s))$ was less than Luesamai and Chongcharoen's (2018) upper bound $(R_0(u,i_s))$ and decreased as n increased and initial surplus u increased. Meanwhile, the lower bound sequence $(\psi_n(u,i_s))$ increased when n increased but decreased when u increased.

Example 2: For all $n = 1,2,3,\ldots$, suppose that the total claim amount during period n is $Y_n \sim N(5,(1.2)^2)$, the bond interest rate at time n is $I_n \in \{0.01, 0.015, 0,02\}$ with transition probability matrix

$$\begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.4 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.2 \end{bmatrix},$$

and $I_0 = 0.015$. Meanwhile, the gross stock return at time n is $W_n = e^{\left(\mu-\frac{\sigma^2}{2}\right)+\sigma B_n}$, where $B_n \sim N(0,1)$, $\mu = 0.9$, and $\sigma = 0.2$. The safety loading amounts given by the insurer (θ) and the reinsurer (δ) were assumed to be 10% and 11%, respectively, and the initial surplus value was set as u =14. The policies of the reinsurance and the amount of investment in stock were assumed to be the same in each time period (i.e., $b_n = b$ and $\alpha_n = \alpha$, for all $n = 1,2,3,\ldots$).

The values of the ruin probability for a finite time period $\psi_n(u, i_s)$ and the sequence of $R_n(u, i_s)$, where $n = 1,2,3$, are reported in Table 1 for $b = 0.6, 0.8, 1$ and $\alpha = 0.05, 0.1, 0.2$.

Table 1: The first three iteratively obtained upper and lower bounds of ultimate ruin probability $\psi(u, i_s)$ are illustrated together with Luesamai and Chongcharoen's (2018) upper bound for varying retention level of reinsurance b and amount of investment in stock α.

b	α	The Proposed Iteratively Obtained Bounds								Luesamai and Chongcharoen's (2018) Upper Bound
		Lower Bound				Upper Bound				
		$\psi_1(u, i_s)$	$\psi_2(u, i_s)$	$\psi_3(u, i_s)$	…	…	$R_3(u, i_s)$	$R_2(u, i_s)$	$R_1(u, i_s)$	
0.6	0.05	2.792e-91	4.583e-74	9.167e-74	…	…	1.20427e-07	1.20425e-07	1.20424e-07	1.85788e-07
	0.1	1.523e-92	4.091e-76	8.183e-76	…	…	9.25554e-08	9.25542e-08	9.25530e-08	1.62847e-07
	0.2	8.982e-95	1.494e-79	2.989e-79	…	…	5.51173e-08	5.51166e-08	5.51159e-08	1.25374e-07
0.8	0.05	3.810e-53	9.604e-51	1.924e-50	…	…	2.49451e-06	2.49447e-06	2.49444e-06	3.57987e-06
	0.1	6.483e-54	3.967e-52	8.000e-52	…	…	2.02816e-06	2.02813e-06	2.02811e-06	3.22808e-06
	0.2	2.469e-55	1.679e-54	3.605e-54	…	…	1.34753e-06	1.34751e-06	1.34749e-06	2.62820e-06
1.0	0.05	2.789e-35	2.790e-35	8.369e-35	…	…	1.21331e-05	1.21329e-05	1.21328e-05	1.66624e-05
	0.1	8.508e-36	8.509e-36	2.552e-35	…	…	1.02342e-05	1.02340e-05	1.02339e-05	1.53051e-05
	0.2	8.974e-37	8.974e-37	2.692e-36	…	…	7.30661e-06	7.30651e-06	7.30642e-06	1.29244e-05

It can be seen that the values of the proposed lower bound $(\psi_n(u, i_s))$ and upper bound $(R_n(u, i_s))$ increased when retention level value b increased and also decreased when the amount of investment in stock α, increased. The other trends support the conclusions drawn for Example 1, where $\psi_n(u, i_s)$ increased and $R_n(u, i_s)$ decreased as n increases and $R_n(u, i_s)$ was lower than Luesamai and Chongcharoen's (2018) upper bound.

4 Conclusions

We applied both Gajek's (2005) and Rudź's (2015) operator approaches to iteratively determine the lower and upper bounds of the ultimate ruin probability, $\psi(u, i_s)$, in a discrete time risk model with proportional reinsurance and investment first presented by Luesamai and Chongcharoen (2018). The algorithm is based on iterating the integral operator L which is defined in (11). The bounds were constructed under the assumption that the retention level of reinsurance and the amount of investment in stock in each time period for the risk model are set as constant values (i.e., $b_n = b$ and $\alpha_n = \alpha$, for all $n = 1,2,3, \ldots$). The ruin probability for a finite time, $\psi_n(u, i_s)$, can be provided in terms of the integral operator L as $\psi_n(u, i_s) = L^{n-1}\,\psi_1(u, i_s)$ where $\psi_1(u, i_s) = \sum_{t=0}^{d} p_{st} \int_0^\infty \bar{G}(\pi) dF(w)$. Since $\psi_n(u, i_s)$ monotonically increases from $\psi_1(u, i_s)$ to $\psi(u, i_s)$ as n increases, $\psi_n(u, i_s)$ can be interpreted as the lower bound of ultimate ruin probability. The upper bound of ultimate ruin probability is $R_n(u, i_s) = L^n\, R_0(u, i_s)$, where $R_0(u, i_s)$ is Luesamai and Chongcharoen's (2018) upper bound. When n increases, $R_n(u, i_s)$ monotonically decreases from $R_0(u, i_s)$ to $\psi(u, i_s)$. Besides, we showed that the ultimate ruin probability is a fixed point of L; i.e., $\psi(u, i_s) = L\psi(u, i_s)$.

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