# Economic Capital Determination for Non-Life Insurance Using Copulas

**Joshua Gmayenaam Makubu**

*University of Ghana Business School, Madina, Ghana*
joshauesmakubu@gmail.com

**Charles Andoh**

*University of Ghana Business School, Madina, Ghana*
candoh@ug.edu.gh

**Lord Mensah**

*University of Ghana Business School, Madina, Ghana*
lmensah@ug.edu.gh

Received: February 8, 2022
Revised: June 7, 2022
Accepted: June 24, 2022

## Abstract

*In the light of the not-too-distant ago financial crisis that brought the interconnectedness of financial risks into focus, this paper set out to evaluate the economic capital of the non-life insurance industry in Ghana by evaluating the total risk exposure of the industry via value at risk and conditional value at risk. The paper employed the copula-based ARMA-GARCH model on quarterly premium, claims and asset data of the Ghanaian non-life insurance industry spanning the period 2012Q4 to 2020Q4. All lines of business in the non-life insurance industry in Ghana were considered and predictions were made for the next 8 quarters of the data. It was found that over the quarters in 2021 and 2022, there is a 5% chance that in each quarter the industry will lose more than 64% of its entire portfolio and that should extreme events trigger such losses then it should be expected that losses will amount to about 80% of the industry portfolio. It was further found that there is a 1% chance in each of the quarters from 2021 to 2022, that the non-life insurance industry's portfolio will be wiped out. Such an event, though as remote as 1 out of 100, can have dire consequences for the economy and financial sector and therefore the non-life insurance industry is one area of the Ghanaian financial sector that must be closely monitored to avert any such catastrophe.*

**Keywords:** Copulas, Conditional Value-at-Risk, Economic Capital, Non-life Insurance, Value-at-Risk

# 1 Introduction

The recent financial crisis has brought into focus the connectedness of market risk, credit risk, and operational risk such that these three risks may act simultaneously to precipitate catastrophic losses if they are not properly managed (Slepov *et al.*, 2019; Shim & Lee, 2017; Mejdoub & Arab, 2017). The increasing complexity of the insurance markets with products that were hitherto unavailable requires that extreme risks need to be properly managed now more than ever if firm sustainability is to be achieved on a going basis (Eling, Nuessle, & Staubli, 2022; Goovaerts, Kaas & Laeven, 2010). A distressed insurer may translate to a distressed client by virtue of the insurer's impaired ability to deliver on insurance promises which in turn may have a greater macroeconomic impact (Slepov *et al.*, 2019; Diers, Eling & Marek, 2012).

An implication of the Solvency II directive is the identification of the overall loss distribution of an insurer as the paradigm shift is now toward economic risk-based approaches to ensuring solvency in financial undertakings (Leković, 2018; Skoglund, 2010). This has engendered the need to manage risk holistically and not in silos thereby requiring aggregation of total risk exposures. To this end, the dependencies between individual lines of risk are very important since it feeds directly into the result of risk aggregation. With the neglect of the exact nature of these dependencies, the insurer may either assume total independence or complete dependence among these risks. The first case will lead to underestimation of the total risk faced by the insurer thus leaving it with inadequate protection against extreme events while the second case will overestimate the total risk of the insurer and may therefore incur too high capital costs (Marti *et al.*, 2021; Bi & Cai, 2019; Lin, Sun, & Yu, 2018; Braun, Schmeiser & Schreiber, 2015; Kretzschmar, McNeil & Kirchner, 2010). The Solvency II draft directive acknowledges this fact and proposes recognition of dependencies by the use of linear correlations but for adequate loss aggregation deriving from various risk classes, and therefore an accurate calculation of total risk capital, existing stochastic dependencies between risk-specific losses have to be adopted by integrated risk management approaches (Marti, *et al.*, 2021; Lin, *et al.*, 2018; Grundke,2010; Kretzschmar, McNeil & Kirchner, 2010).

In compliance with the International Association of Insurance Supervisors (IAIS), at any point in time, the assets of an insurance company should be 150% of its liabilities or its net assets should be at least equal to the minimum capital requirement for solvency to hold (Mukhtarov, Schoute, & Wielhouwer, 2022). The regulator of the Ghanaian insurance industry, the National Insurance Commission, has recently more than tripled the regulatory capital required of insurers from GH¢15million to GH¢50million effective the end of June 2021 (NIC, 2021).

This paper, therefore, aims to present a model for evaluating the economic capital of the insurance industry in Ghana and specifically investigate the economic capital of the non-life insurance industry to provide a basis of comparison to the regulatory capital requirement.

In so doing, this paper contributes to the extant literature on the modeling of non-linear dependencies with application to the non-life insurance industry in the aggregation of interdependent risks using copulas.

In the following section of this paper, economic capital, value-at-risk (VaR), and copulas are explained. This is followed by a section explaining the methodology used in the paper followed by a section presenting and discussing the results of this study. A final concluding section then follows with the conclusions and recommendations from this study.

# 2 Literature Review

The recent financial crisis brought to the fore the interconnectedness of risks and has engendered an upsurge in the integrated risk management process that involves a holistic view of aggregate risks. One area that is receiving attention is the non-linear dependencies of the financial risks and the modeling of these dependencies using copulas (see Araichi, & Almulhim, 2021; Mejoub & Ben Arab, 2017). This section reviews the empirical literature relevant to using copulas in the estimation of risk capital.

## 2.1 Economic Capital and VAR

Economic capital for an insurer is that extra financial capacity required to cushion its insurance business in the event that underwriting losses exceed expectations or returns on investments fall below expectations, such that, even in such an adverse condition the insurer will still be able to continue its business (Furman, Hackman, & Kuznetsov, 2020; Tang & Valdez, 2009). Thus, economic capital is supposed to be a “rainy day fund, so when bad things happen, there is money to cover it.” In this vein, Kretzschmar, McNeil, and Kirchner (2010) note that “economic capital is the amount of capital required by a financial firm in order to function as a solvent entity at a stated confidence level over a given time period considering the risk profile of the firm” (Kretzschmar, McNeil & Kirchner, 2010).

Whereas regulatory capital is the minimum capital required by regulation to be kept by insurers, economic capital on the other hand embodies all the actual risk exposures of the insurer and as Shim and Lee put it “because economic capital is the level of capital that the firm should hold to maintain its probability of default below a certain threshold, economic capital can be viewed as the Value-at-Risk (VaR) at a given confidence level” (Yan, Zhengyin, & Yuna, 2019; Shim & Lee, 2017).

At a given confidence level, VaR reports how much is required to be able to manage risk exposure of a portfolio over a specified time horizon thus when the VaR indicates that over the next time horizon there is an alpha probability that loses will exceed the VaR, it means there is a 1-alpha confidence that the VaR is all that is needed to manage loses over the next time horizon (Hasnaoui, 2018).

VaR has taken preeminence in the quantification of risk especially in financial risk management by both practitioners and regulators in recent times (Omari & Mwita, 2018). It is a quantile measure that defines the maximum loss due to a change in asset value over a given period with a given confidence level. VaR is easy to compute for single variables such as the return on a single asset but becomes increasingly complex when multiple asset portfolios are considered due to the dependency of asset distributions to consider in evaluating the VaR.

For a portfolio made up of multiple assets, VaR can be mathematically defined as

$$VaR_q(L) = \inf\{l \in R: P(L > l) \leq 1 - q \} = \inf\{l \in R: F_L(l) \geq q\}$$

where $q \in (0,1)$ is a given confidence level, $L$ is the portfolio loss and $l$ is the smallest number such that the probability that $L$ exceeds $l$ is no greater than $1 - q$ (Omari & Mwita, 2018). Thus, in its simplest form, VaR is the q-quantile of the loss distribution data if such data is readily available.

VaR of an $n$-asset portfolio can be evaluated by the traditional variance-covariance method as

$$\sigma_{p,t}^2 = [w_1 \ldots w_n] \begin{bmatrix} \sigma_{1,t}^2 & \cdots & \sigma_{1n,t} \\ \vdots & \ddots & \vdots \\ \sigma_{n1,t} & \cdots & \sigma_{n,t}^2 \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$

$$VaR_{p,t}(\alpha) = \sigma_{p,t}.Z_\alpha + \mu_{p,t}$$

where $\mu_{p,t}$is portfolio return at time $t$ and $\sigma_{p,t}^2$is variance of portfolio returns at time $t$ but this formula assumes the assets are multivariate normal and linearly correlated (Huang, Lee, Liang & Lin, 2009).

The reality of fatter tails stylized fact in financial time series violates the linear correlation assumptions employed in the traditional models of evaluating the VaR and has thus been shown in empirical literature that the assumption of linear correlation does not provide adequate results when evaluating VaR due to stylized facts of financial time-series such as excess kurtosis, asymmetry, and leverage among others since VaR is mostly concerned with the tails of the distributions (Arif *et al.*, 2021; Chen, Nguyen, & Stadie, 2018; Nguyen & Molinari, 2011).

A risk metric related to the VaR is the conditional value-at-risk (CVaR). The CVaR indicates the expectation of losses in case the loss experience goes beyond the VaR. That is

$$CVaR_q(L) = E\left[L: L \geq VaR_q(L)\right]$$

The correlations between asset returns have been found to be dynamic and not static, with higher correlation observed during volatile market periods and downturns in the market than during calm market conditions (Chen, Nguyen, & Stadje, 2018). This goes to show that extreme risk events may occur with a higher correlation between asset returns than normal risk events and such asymmetries cannot be properly modeled by symmetric distributions such as the Gaussian (Arif *et al.*, 2021; Embrechts, McNeil & Straumann, 2002), therefore, aggregating risks with linearity and normality assumptions will produce inadequate aggregated risk estimates.

To overcome these problems, this paper resorts to the copula theory which allows for the construction of flexible multivariate distributions with different marginals and different dependency structures allowing the joint distribution of a portfolio to be free from any normality and linear correlation assumptions.

## 2.2 Copulas

Copulas have recently become a most significant new tool in the field of finance in terms of risk management, portfolio allocation, and derivative asset pricing, among others. A copula is a function with a specific dependence structure that connects a joint distribution to univariate marginals irrespective of the individual marginal distributions. This enables the creation of a probability distribution to model dependent marginal distributions. The dependence measures derived from copulas can overcome the shortcomings of the linearity assumptions in the traditional risk aggregation techniques and have broader applications. This is because copulas can be used to describe more complex multivariate dependence structures, such as non-linear tail dependence (Puccetti, 2019; Eling, & Jung, 2018).

Haugh (2016) defines a d-dimensional copula, C: $[0, 1]^{d}: \rightarrow [0, 1]$ as a cumulative distribution function (CDF) with uniform marginals and according to Sklar (1959) theorem if we consider a $d$-dimensional CDF, $F$ , with marginals $F_1, \dots, F_d$, there exists a copula, $C$, such that

$$F(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d)) \; \forall \; x_i \in [-\infty, \infty] \; \wedge \; i = 1, \dots, d.$$

Now if we assume that $X_i$ is a random return variable that has a marginal distribution

$$F_i(X) = P(X_i \leq x)$$

and let $\mu_i$ be the observed value of $F_i(X_i)$ then for continuous univariate marginals, the unique copula function is given by

$$C(\mu_1, \dots, \mu_n) = F(F_1^{-1}(\mu_1), \dots, F_n^{-1}(\mu_n))$$

where $F_1^{-1}, \dots, F_n^{-1}$ are the quantile functions of the univariate marginals, $F_1, \dots, F_n$.

Empirical studies using copulas note that tail dependencies in financial literature has been shown to be better modelled by copulas instead of correlations (see Rosenberg & Schureman, 2006) but at times the choice of the copula to use may be difficult as noted by Grundke (2010) who in assessing the accuracy of economic capital based on the top-down approach, noted that it was relatively difficult to select a copula function that captures the risk aggregated by the bottom-up approach. Liang, *et al.*, (2013) compared factor, elliptical and Archimedean copulas in their work on credit risk and market risk integration of Chinese banks and found that factor copulas led to a more cautious risk aggregation. Li, *et al.*, (2015) also did a comparative copula study when they reviewed extant methods of aggregating market, credit and operational risks using the Austrian banking industry and found the t-copula to be an adequate method for capturing tail dependence while the Gaussian copula was not recommended. Guharay, Chang, and Xu (2018) noted that the VaR risk metric has always been difficult to robustly estimate for different data types especially as the classical approaches assumes independence of loss severity and frequency (which they contend is not always the case in real life situations). Thus, Guharay, Chang, and Xu (2018) employed a mixture copula-based approach to robustly estimate VaR in heavy tail data.

# 3 Methodology

## 3.1 Data

Quarterly premiums and claims data of the non-life industry comprising 29 insurance firms from 2012Q4 to 2020Q4 was collected on all lines of business of each insurer, that is:

- Fire: - for each insurance company the premiums collected for Fire cover as well as claims paid on Fire policies in each quarter.
- Motor: - for each insurance company the premiums collected for Motor cover as well as claims paid on Motor policies in each quarter.
- Personal Accident: - for each insurance company the premiums collected for Personal Accident cover as well as claims paid on Personal Accident policies in each quarter.
- Marine and Aviation: - for each insurance company the premiums collected for Marine and Aviation cover as well as claims paid on Marine and Aviation policies in each quarter.
- Liability: - for each insurance company the premiums collected for Liability cover as well as claims paid on Liability policies in each quarter.
- Bond: - for each insurance company the premiums collected for Bond cover as well as claims paid on Bond policies in each quarter.
- Others: these are lines of business that are infrequent (sometimes one-off policies), for example, cover for a specific engineering project.

*Ghana Stock Exchange Composite Index Level: this data is collected as a proxy for the general performance of equity in the Ghanaian economy.*

*91-day Treasury Bill Rate: this data is used as a proxy for the general performance of bonds in the Ghanaian economy.*

### 3.2 Data Transformation

For each quarter and each line of business the data is transformed to return series using the formula:

$$Underwriting\ return = \frac{Premium - Present\ Value\ of\ Claims}{Premium}$$

and for the asset series, the continuous returns are evaluated for the GSE-CI while the government bond is scaled to reflect the quarterly returns as shown below.

$$Stock\ returns = \frac{\log GSECI\ current\ level}{\log GSECI\ previous\ level}$$

$$Bond\ returns = \frac{91 - day\ T - bill\ rate}{4}$$

### 3.3 Copula based ARMA-GARCH Model

To account for the stylized facts of financial time series such as conditional heteroscedasticity, heavy tails and other conditional dependencies that may impact the economic capital estimation, we employ the copula based ARMA-GARCH model to model our data.

#### 3.3.1 Marginal Distributions

The assumption of normality that underlies the classical theoretical developments in financial time series are often violated in real datasets (Puccetti, 2019). As an example, using our data, we illustrate with a normal quantile-quantile plot from our dataset as shown in Figure 1 below. The data is from the underwriting returns of the fire line of business and the bond investment returns. In a normal quantile-quantile plot, if the data is from the normal distribution, the circles will plot close to the straight line, but it is observed that is not the case in each of the two plots in Figure 1 indicating that the fire underwriting and bond investment returns are not from the normal distribution. If Figure 2 (right), the fire underwriting density plot shows skewness and a heavy tail indicating that the use of a symmetric distribution that does not capture tail dependencies (like the normal distribution) will underestimate the risk exposure inherent in the series (Punzo, Bagnato, & Maruotti, 2018).

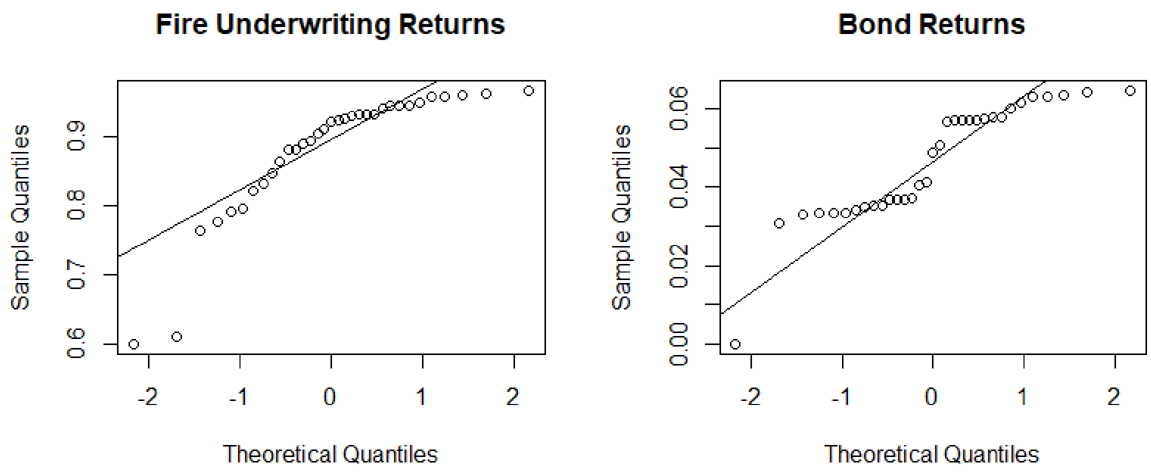



Figure 1: *Normal Q-Q Plots for Fire Underwriting and Bond Returns*

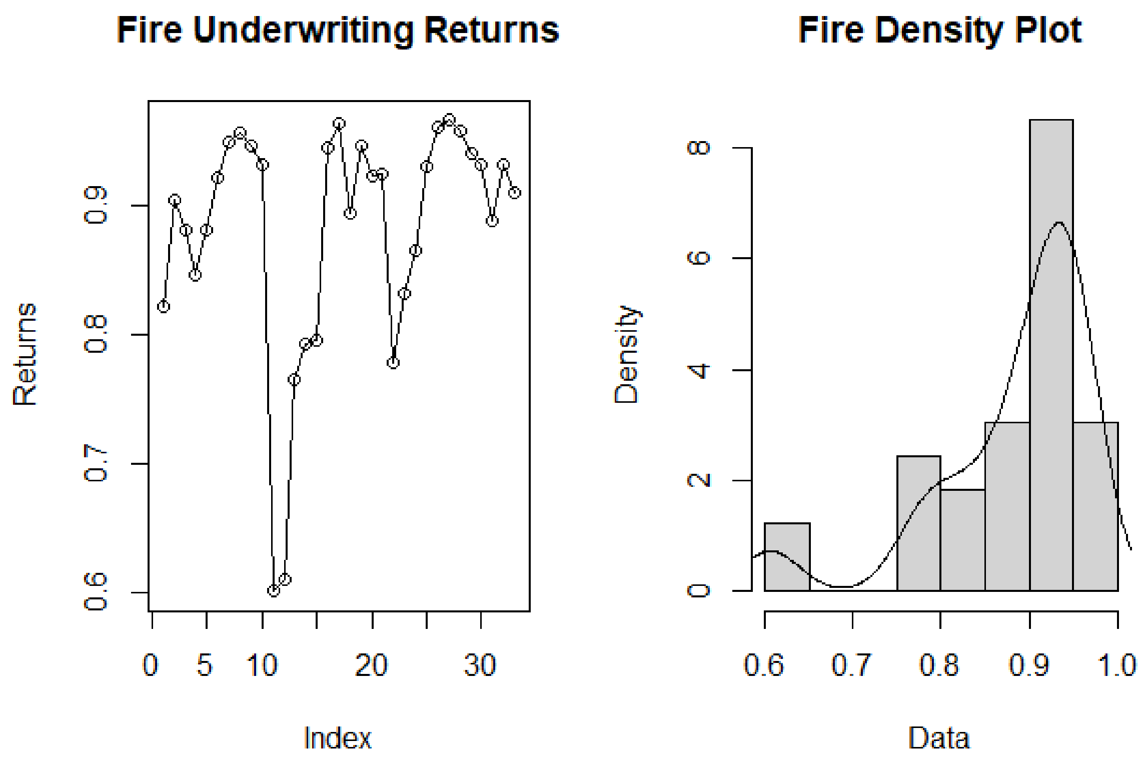



Figure 2: *Underwriting Returns Volatility and Density Plot for Fire Line of Business*

To identify the underlying marginal distributions of the data for the study, each variable was fitted with several distributions and the best fitting distribution chosen based on loglikelihood and information criteria. Table 1 below shows the marginal distributions closest to the empirical distributions of the variables as well as associated parameters. The choice of distributions fitted were based on actuarial literature. The distributions tested for each variable are Beta, Birnbaum-Saunders, Exponential, Extreme Value, Gamma, Generalized Extreme Value, Generalized Pareto, Inverse Gaussian, Logistic, Log-Logistic, Lognormal, Nakagami, Normal, Rayleigh, Rician, t-Location-Scale and Weibull. To illustrate, we present Figure 3 as an example of the process of choosing a close matching distribution for each variable. In the figure, the stock returns variable is shown, with the best four fitting distributions superimposed.

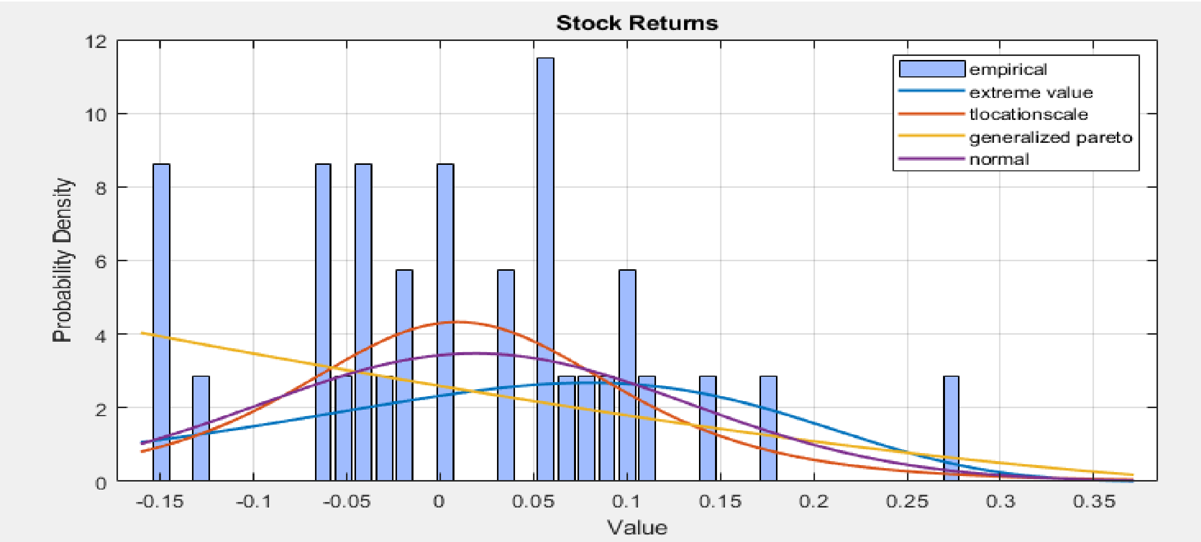



Figure 3: *Distributions fitting for stock returns showing the 4 best fitting distributions.*

Table 1: Marginal Distributions of Data

| | Distribution | NLogL | BIC | AIC | Distribution parameters | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | mu | sigma | k | theta |
| **Fire** | GPD | -52.10 | -93.71 | -98.20 | | 0.53 | -1.45 | 0.60 |
| **Motor** | GPD | -52.89 | -95.29 | -99.78 | | 0.13 | -0.60 | 0.61 |
| **Accident** | EV | -36.29 | -65.58 | -68.57 | 0.82 | 0.06 | | |
| **MnA** | EV | -51.65 | -96.31 | -99.30 | 0.99 | 0.04 | | |
| **Liability** | Logistic | -34.13 | -61.28 | -64.27 | 0.87 | 0.05 | | |
| **Bonds** | GPD | -34.21 | -57.93 | -62.41 | | 0.50 | -1.17 | 0.57 |
| **Others** | GPD | -726.14 | -1441.79 | -1446.28 | | 0.62 | -6.90 | 0.91 |
| **Stock** | EV | -17.88 | -28.76 | -31.75 | 0.08 | 0.14 | | |
| **Gbond** | GPD | -99.52 | -188.55 | -193.03 | | 0.09 | -1.37 | 0.00 |

*Notes: GPD (Generalized Pareto Distribution); EV (Extreme Value distribution); NLogL (Negative Loglikelihood).*

### 3.3.2 ARMA-GARCH Model

To deal with the stylized issue of volatility clustering in financial time series as well as skewness and heavy tails, we adopt the skewed Student's t-distributed ARMA(1,1)-

GARCH(1,1) model. This is a mean-variance model with the ARMA(1,1) part modelling the conditional mean process while the GARCH(1,1) models the conditional time-varying volatility process. The order of the model was selected from testing several ARMA-GARCH models and selecting the best fitting model using log-likelihood, information criteria as well as Ljung-Box tests. The model is given as

$$X_t = \mu + \phi(X_{t-1} - \mu) + \theta \varepsilon_{t-1} + \varepsilon_t$$

$$\varepsilon_t = \sigma_t \eta_t$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

A pairwise scatter plot of the variables clearly shows that the dependencies may not be linear as illustrated in Figure 4 below (see Appendix for other plots). It is observed that there is no discernible pattern in the pairwise plots, we therefore adopt the $t$-copula, which is implied by the multivariate Student's t-distribution, to investigate the global dependence structure of our data.

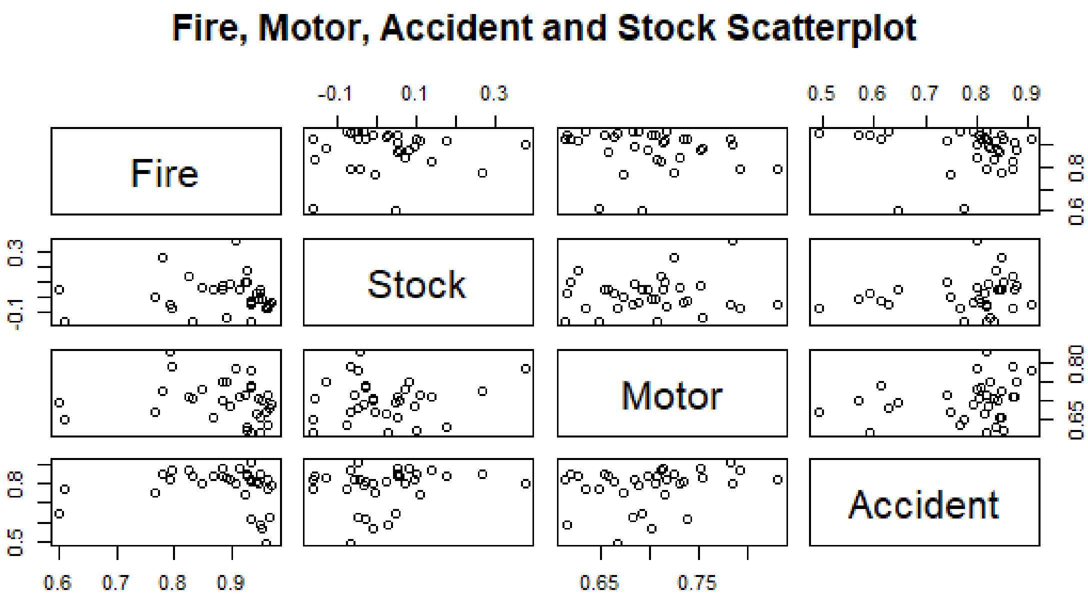



Figure 4: *Dependency Among Variables*

The $t$-copula with $\nu$ degrees of freedom can be written as

$$C_{\nu,\rho}^{t}(\mu_1, \dots, \mu_n) = t_{\nu,\rho}(t_\nu^{-1}(\mu_1), \dots, t_\nu^{-1}(\mu_n))$$

where $t_{\nu,\rho}$ is the multivariate $t$-distribution function with correlation matrix $\rho$ and $\nu$ degrees of freedom while $t_\nu$ is the marginal $t$-distribution function.

Huang and Shemyakin (2020) have noted that t-copulas have recently become popular as a modeling tool of non-linear dependence in statistics. The preference of the $t$-copula is its ability to generate joint extreme movements regardless of the marginal behaviour of the individual random variables while at the same time incorporating non-normal characteristics of stochastic variables. This is a desirable feature in cases where extreme events can occur simultaneously which is what the case should be when aggregating risks in an economic capital modelling situation. Zeevi and Mashal (2002) showing that equity market indices exhibited extremal behaviour found that the $t$-copula appropriately represented the dependence structure of extreme co-movements of financial asset returns. Breymann, Dias and Embrechts (2003) also showed that the $t$-copula empirically provides the best fit for financial returns data while Brechmann, Czado and Paterlini (2014) also found the $t$-copula to provide a good fit for operational risk losses. Huang and Shemyakin (2020) compared skewed t-copula models for insurance and financial data and suggested the Metropolis-Hastings algorithm with block updates to deal with the problem of intractability of conditionals in skewed t-copulas.

Some copulas like the Clayton copula are best suited for modelling lower tailed distributions while others like the Gumbel copula are for upper tailed distributions but the t-copula can model both upper and lower tailed distributions and looking at Figure 5 below it is observed that the distributions we are aggregating are characterized by both upper and lower tails hence providing further support of the choice of the multivariate t-copula as the copula of choice.

### 3.3.3 Steps in Economic Capital Estimation Procedure

Step 1: Standardize residuals from ARMA-GARCH Model.
Step 2: Convert standardized residuals to Uniform (0,1) samples.
Step 3: Use Student's t-copula to generate 200,000 realizations of $U_i,\ i = 1, \dots, 9$ where the $U_i'$s represent the 2 asset and 7 liability lines.
Step 4: Obtain $F_i^{-1}(U_i)$ to transform the uniformly distributed values back into the original units.
Step 5: Let $R_i = F_i^{-1}(U_i)$ and define $A_t = w_1 R_{1,t} + \cdots + w_9 R_{9,t}$ where $w_i's$ are the weights of the 9 data lines.
Step 6: Estimate $VaR_{\boldsymbol{\alpha}}(t)$ and $CVaR_{\boldsymbol{\alpha}}(t)$ at time $t$ at a significance level $\boldsymbol{\alpha}$. $VaR_{\boldsymbol{\alpha}}$ is the $\boldsymbol{\alpha}$-quantile of the weighted distribution $A_t$.

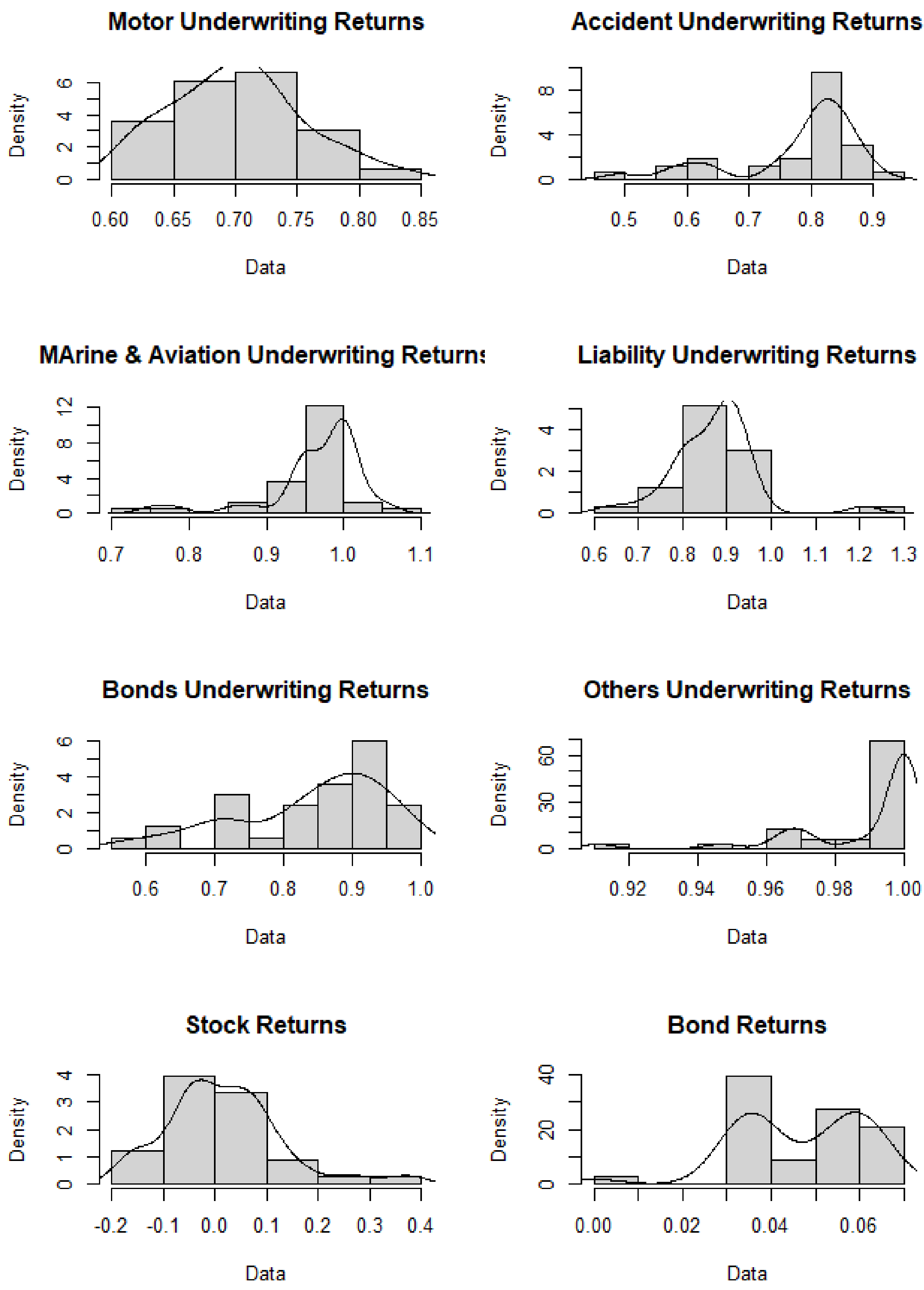



Figure 5: *Skewness and Tails of Marginal Distributions*

# 4 Data Analysis

Figure 6 and Table 2 below gives some perspectives on the insurance industry in Ghana over the period of the study. It is observed from Figure 4 that the Motor line of business is predominant making up 47% of overall industry premiums written over the period of study with the Fire line of business taking second place at 23% of the industry. This shows that Motor and Fire constitutes 70% of the entire insurance business in Ghana as measured by gross premiums written.

Thus, these two lines of business may pose the largest source of underwriting risk exposure for the insurer but over the period of study, underwriting returns for all lines of insurance business have been above 70% (see Table 2 below).

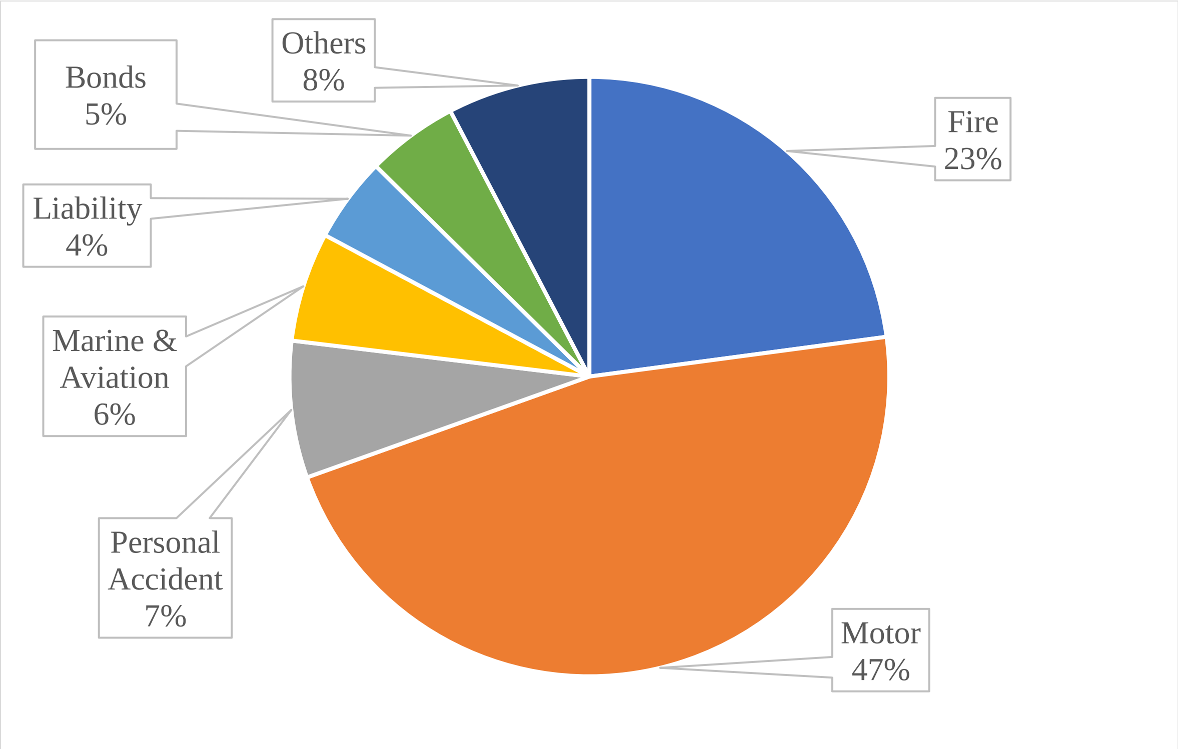



Figure 6: *Line of business share of market by premiums written*

The line of business grouped under others contains insurance for engineering and other one-off insurance arrangements and the data shows that there are instances for this group where premiums were paid, and no claims were made for the quarter therefore translating into a 100% return on underwriting hence this group is observed to have the highest mean underwriting return (Table 2) even though it forms only 8% of the market. Motor line of business had the lowest mean underwriting returns (of 70%) even though it constitutes the largest chunk of the market, and being almost half of the market, this indicates that the exposure from Motor in any quarter can significantly impact the entire industry.

Table 2: Summary Statistics

| | Mean | Standard Deviation | Minimum | Maximum |
|---|---|---|---|---|
| Fire | 0.8822 | 0.0917 | 0.6010 | 0.9668 |
| Motor | 0.7006 | 0.0532 | 0.6149 | 0.8308 |
| Personal Accident | 0.7819 | 0.1011 | 0.4914 | 0.9075 |
| Marine and Aviation | 0.9642 | 0.0647 | 0.7499 | 1.0556 |
| Liability | 0.8697 | 0.0950 | 0.6366 | 1.2103 |
| Bonds | 0.8413 | 0.1091 | 0.5705 | 0.9945 |
| Others | 0.9892 | 0.0204 | 0.9108 | 1.0000 |
| Stock | 0.0188 | 0.1147 | -0.1604 | 0.3713 |
| Gov Bond | 0.0474 | 0.0123 | 0.0308 | 0.0646 |

The asset side of the data shows mean quarterly equity returns (1.88%) being lower than bond returns (4.74%), but it is observed that the equity returns ranges from -16.04% to as high as 37.13% while the bond returns ranged from 3.08% to 6.46% showing equity to be more volatile than bonds as expected from literature.

## 4.1 ARMA-GARCH Modelling

Similar to Shim and Lee (2017), different GARCH(p, q) models ($p = 1,2 \ and \ q \in 1,2$) were fitted to the data to model conditional heteroscedasticity of the returns and it was found that in most cases the GARCH(1,1) model fitted better and hence for model parsimony and in tandem with financial timeseries literature (Bera & Higgins, 1993), the GARCH(1,1) was chosen to model the variance process. Table 3 below shows the loglikelihood, Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) values for the models tested prior to choosing the parsimonious GARCH (1,1) for the variance process. A model is preferred over another if it minimizes information criteria. It is therefore observed from Table 3 that the AIC chose GARCH (1,2) for the underwriting returns of Fire, but BIC chose GARCH (1,1). In case such as this, the choice of BIC is preferred since AIC is known to overestimate model size in some cases. For the variables Motor, Liability, Bonds, Others, Stock and Gbond both AIC and BIC chose GARCH (1,1) while for Accident both AIC and BIC chose GARCH (1,2) and for Marine and Aviation both AIC and BIC chose GARCH (2,1).

Table 3: GARCH Model Selection Criteria

| **Variable** | | | **GARCH (1,1)** | **GARCH (1,2)** | **GARCH (2,1)** | **GARCH (2,2)** |
|---|---|---|---|---|---|---|
| **Fire** | Model Selection Criteria | LL | 50.12985 | 51.41812 | 50.13081 | 51.41812 |
| | | AIC | -2.6139 | -2.6314 | -2.5534 | -2.5708 |
| | | BIC | -2.2965 | -2.2686 | -2.1906 | -2.1627 |
| | Ljung-Box Test | P-value1 | 0.8431 | 0.942 | 0.8404 | 0.942 |
| | | P-value5 | 1 | 1 | 1 | 1 |
| **Motor** | | LL | 54.48723 | 54.48636 | 54.48636 | 54.48636 |
| | | AIC | -2.878 | -2.8174 | -2.8174 | -2.7567 |

| Variable | | | GARCH (1,1) | GARCH (1,2) | GARCH (2,1) | GARCH (2,2) |
|---|---|---|---|---|---|---|
| | Model Selection Criteria | BIC | -2.5606 | -2.4546 | -2.4546 | -2.3486 |
| | Ljung-Box Test | P-value1 | 0.800264 | 0.801271 | 0.801355 | 0.801285 |
| | | P-value5 | 0.00709 | 0.007229 | 0.007236 | 0.007229 |
| **Accident** | Model Selection Criteria | LL | 41.3803 | 44.4413 | 41.43266 | 44.44081 |
| | | AIC | -2.0837 | -2.2086 | -2.0262 | -2.1479 |
| | | BIC | -1.7662 | -1.8458 | -1.6634 | -1.7398 |
| | Ljung-Box Test | P-value1 | 0.3346 | 0.7339 | 0.3415 | 0.7342 |
| | | P-value5 | 0.9893 | 0.9756 | 0.9898 | 0.9757 |
| **MnA** | Model Selection Criteria | LL | 88.30441 | 90.33503 | 94.08321 | 86.4274 |
| | | AIC | -4.9275 | -4.99 | -5.2172 | -4.6926 |
| | | BIC | -4.6101 | -4.627 | -4.8544 | -4.2844 |
| | Ljung-Box Test | P-value1 | 0.8467 | 0.846 | 0.8467 | 0.847 |
| | | P-value5 | 1 | 1 | 1 | 1 |
| **Liability** | Model Selection Criteria | LL | 39.28427 | 40.57284 | 39.47852 | 40.57284 |
| | | AIC | -1.9766 | -1.9541 | -1.9078 | -1.9135 |
| | | BIC | -1.6392 | -1.6113 | -1.545 | -1.5054 |
| | Ljung-Box Test | P-value1 | 0.2795 | 0.1421 | 0.2686 | 0.1422 |
| | | P-value5 | 0.9916 | 0.4942 | 0.9889 | 0.4943 |
| **Bonds** | Model Selection Criteria | LL | 28.67468 | 28.7015 | 28.70193 | 28.70166 |
| | | AIC | -1.31362 | -1.25464 | -1.25466 | -1.194 |
| | | BIC | -0.99618 | -0.89185 | -0.89187 | -0.7859 |
| | Ljung-Box Test | P-value1 | 0.9776 | 0.9795 | 0.9807 | 0.9809 |
| | | P-value5 | 0.9996 | 0.9996 | 0.9996 | 0.9996 |
| **Others** | Model Selection Criteria | LL | 271.3373 | | 268.2912 | |
| | | AIC | -16.02 | | -15.775 | |
| | | BIC | -15.703 | | -15.412 | |
| | Ljung-Box Test | P-value1 | 0.00803 | | 0.01695 | |
| | | P-value5 | 0 | | 0 | |
| **Stock** | Model Selection Criteria | LL | 28.47423 | 28.47559 | 28.47559 | 28.47559 |
| | | AIC | -1.30147 | -1.24094 | -1.24094 | -1.1803 |
| | | BIC | -0.98403 | -0.87816 | -0.87816 | -0.7722 |
| | Ljung-Box Test | P-value1 | 0.8019 | 0.8065 | 0.8065 | 0.8065 |
| | | P-value5 | 0.94 | 0.9435 | 0.9435 | 0.9435 |
| **Gbond** | | LL | 85.86294 | 85.84529 | 85.84494 | 85.84529 |
| | | AIC | -4.7796 | -4.7179 | -4.7179 | -4.6573 |

| Variable | | | GARCH (1,1) | GARCH (1,2) | GARCH (2,1) | GARCH (2,2) |
|---|---|---|---|---|---|---|
| | Model Selection Criteria | BIC | -4.4621 | -4.3551 | -4.3551 | -4.2492 |
| | Ljung-Box Test | P-value1 | 0.006828 | 0.006742 | 0.006708 | 0.006742 |
| | | P-value5 | 0 | 0.07871 | 0 | 0.07871 |

The estimated parameter values of the ARMA(1,1)-GARCH(1,1) are as shown in Table 4 below. The fit of the model for each of the variables is generally good as evidenced by p-values greater than 5% indicating that we fail to reject the null hypothesis that the distributional assumptions of the innovations are correctly specified at the 5% level of significance testing. This indicates that the student's t-skewed distribution assumption on the innovations is adequate to model the series.

Table 4: Estimated Parameters of ARMA(1,1)-GARCH(1,1) Model

| Variables | μ | ϕ | θ | ω | α | β | Shape | p value |
|---|---|---|---|---|---|---|---|---|
| **Fire** | 0.9253 | 0.5435 | 0.0836 | 0.0004 | 0.1195 | 0.7428 | 2.6336 | 0.9302 |
| **Motor** | 0.6911 | 0.1633 | 0.405 | 0.0002 | 0 | 0.9112 | 5.1622 | 0.1362 |
| **Accident** | 0.8256 | 0.7501 | -0.2152 | 0 | 0 | 0.9794 | 4.7847 | 0.6512 |
| **MnA** | 1 | 0.9994 | -0.1056 | 0 | 0.9278 | 0.0682 | 2.3471 | 0.0001 |
| **Liability** | 0.872 | -0.334 | 0.6304 | 0.0009 | 0 | 0.8173 | 3.493 | 0.4918 |
| **Bonds** | 0.8446 | 0.174 | 0.1629 | 0.0001 | 0 | 0.999 | 95.761 | 0.1159 |
| **Others** | 1 | 0.9656 | -0.0518 | 0 | 0.107 | 0.1856 | 2.3046 | 0 |
| **Stock** | 0.0169 | 0.699 | -0.4815 | 0.0001 | 0 | 0.9977 | 3.1962 | 0.9764 |
| **Gbond** | 0.2308 | 0.9881 | -0.0207 | 0 | 0 | 0.9556 | 2.1 | 0.0061 |

## 4.2 Economic Capital Estimation Results

Table 5: VAR Forecasts for Next 8 Quarters

| | 2021Q1 | 2021Q2 | 2021Q3 | 2021Q4 | 2022Q1 | 2022Q2 | 2022Q3 | 2022Q4 |
|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{VaR_{5\%}}$ | -0.6380 | -0.6372 | -0.6399 | -0.6370 | -0.6388 | -0.6367 | -0.6370 | -0.6381 |
| $\boldsymbol{VaR_{2.5\%}}$ | -0.7625 | -0.7617 | -0.7644 | -0.7616 | -0.7634 | -0.7613 | -0.7616 | -0.7626 |
| $\boldsymbol{VaR_{1\%}}$ | -0.9091 | -0.9083 | -0.9111 | -0.9082 | -0.9099 | -0.9079 | -0.9082 | -0.9092 |

The interpretation of the $\boldsymbol{VaR_{5\%}}$ in 2021Q1 being -0.6380 (see Table 5 above) is that in the first quarter of 2021 there is a 5% chance that industry loses will exceed 63.8% of the entire portfolio of the insurance industry. This is an extreme event indicator, thus, to ensure survival of extreme risk events the insurance industry over this period need 63.8% of its portfolio as risk capital to be 95% confident that should such an event occur, the industry will survive and continue with business as usual. The $\boldsymbol{VaR_{5\%}}$ for the rest of the periods are around this figure hence it can be inferred that for each of the quarters from 2021Q1 to 2022Q4, the insurance industry needs about 64% of its entire portfolio as economic capital to stave off any catastrophic risk event with 95% confidence.

From Table 5, the forecasts show that to be 99% confident in regard to being solvent in case of any catastrophic risk exposure materialization then the industry will need about 91% of its entire portfolio as economic capital. This is derived from the $\boldsymbol{VaR_{1\%}}$ forecasts over the forecast horizon.

For a 97.5% confidence that in the event of extreme risk event, the insurance industry will be solvent to carry on business as usual, the $\boldsymbol{VaR_{2.5\%}}$ forecasts over the forecast horizon indicates that for each of the forecasted quarters, the industry will require about 76% of its entire portfolio.

The VaR risk measure only tells how much is needed at a particular confidence level to mitigate an extreme loss event but the CVaR gives the expected amount that will be lost should such extreme events occur, therefore CVaR measure is computed and shown in the table below.

It is inferred from Table 6 below that there is a 5% chance that over the 2021Q1 the insurance industry will lose 79.98% of its portfolio should extreme loss event occur. This loss amount is roughly the same over the remaining forecast periods hence for each of the forecasted quarters the industry may lose about 80% of its portfolio. This figure goes up to about 90% with a probability of 2.5% as evidenced by the $CVaR_{2.5\%}$ values. There is however a 1% chance that extreme loss events can completely wipe out the portfolio of the insurance industry. This is shown by the $CVaR_{1\%}$ values.

Table 6: CVAR Forecasts for Next 8 Quarters

| | 2021Q1 | 2021Q2 | 2021Q3 | 2021Q4 | 2022Q1 | 2022Q2 | 2022Q3 | 2022Q4 |
|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{CVaR_{5\%}}$ | -0.7998 | -0.7987 | -0.8021 | -0.7985 | -0.8008 | -0.7981 | -0.7985 | -0.7999 |
| $\boldsymbol{CVaR_{2.5\%}}$ | -0.9088 | -0.9078 | -0.9111 | -0.9077 | -0.9098 | -0.9073 | -0.9077 | -0.9089 |
| $\boldsymbol{CVaR_{1\%}}$ | -1.0434 | -1.0425 | -1.0457 | -1.0424 | -1.0443 | -1.0420 | -1.0424 | -1.0436 |

# 5 Summary, Conclusion and Recommendations

This paper has evaluated the economic capital of the non-life insurance industry in Ghana and found that over the quarters in 2021 and 2022, there is a 5% chance that in each quarter the industry will lose more than 64% of its portfolio and that should extreme events trigger such losses then it should be expected that losses will amount to about 80% of the industry portfolio. The average quarterly premium of the entire industry is GH¢701m, thus if the 29 non-life firms surveyed each hold the required minimum capital of GH¢50m, the industry will have enough (GH¢1,450m) to manage such an exposure. But it was observed that the total industry premium for the last quarter in 2020 was highest, recording GH¢1,857 and so the total industry regulatory capital will just be enough at the 5% to cover its exposure. This means that as the non-life industry underwrites more risks (as evidenced by increasing premium collections) the regulator must consistently update and increase the required capital to ensure firm sustainability in the industry.

The research of Denkowska and Wanat (2020) confirms the systemic risk generated by insurance firms and the higher positive correlation among them during global financial turbulence hence the regulator must keep track of the growth in the industry to ensure the required minimum capital is in place to avert disaster.

Since extremal activities have the potential of damaging the industry, this paper has found that there is a 1% chance in each of the quarters from 2021 to 2022, that the non-life insurance industry will be wiped out. Such an event, though as remote as 1 out of 100, can have dire consequences for the economy and financial sector.

Further research can investigate dynamic models that incorporate industry growth to set minimum capital requirements over specified time horizons taking projected industry risk profile into consideration.

# References

Araichi, S., & Almulhim, T. (2021). Vine copulas and fuzzy inference to evaluate the solvency capital requirement of multivariate dependent risks. *Applied Economics*, *53*(52), 6058-6074.

Arif, M., Khan, D. M., Khosa, S. K., Aamir, M., Aslam, A., Ahmad, Z., & Gao, W. (2021). Modelling insurance losses with a new family of heavy-tailed distributions. *Computers, Materials & Continua*, *66*(1), 537-550.

Bera, A. K., & Higgins, M. L. (1993). ARCH models: properties, estimation and testing. *Journal of economic surveys*, *7*(4), 305-366.

Bi, J., & Cai, J. (2019). Optimal investment–reinsurance strategies with state dependent risk aversion and VaR constraints in correlated markets. *Insurance: Mathematics and Economics*, *85*, 1-14.

Braun, A., Schmeiser, H., & Schreiber, F. (2015). Solvency II's market risk standard formula: how credible is the proclaimed ruin probability? *Journal of insurance issues*, 30(1), 1-30.

Brechmann, E., Czado, C., & Paterlini, S. (2014). Flexible dependence modeling of operational risk losses and its impact on total capital requirements. *Journal of Banking & Finance*, *40*, 271-285.

Chen, A., Nguyen, T., & Stadje, M. (2018). Optimal investment under VaR-regulation and minimum insurance. *Insurance: Mathematics and Economics*, *79*, 194-209.

Denkowska, A., & Wanat, S. (2020). Dependencies and systemic risk in the European insurance sector: New evidence based on Copula-DCC-GARCH model and selected clustering methods. *Entrepreneurial Business and Economics Review*, *8*(4), 7-27.

Diers, D., Eling, M., & Marek, S. D. (2012). Dependence modeling in non-life insurance using the Bernstein copula. *Insurance: Mathematics and Economics*, *50*(3), 430-436.

Eling, M., & Jung, K. (2018). Copula approaches for modeling cross-sectional dependence of data breach losses. *Insurance: Mathematics and Economics*, *82*, 167-180.

Eling, M., Nuessle, D., & Staubli, J. (2022). The impact of artificial intelligence along the insurance value chain and on the insurability of risks. *The Geneva Papers on Risk and Insurance-Issues and Practice*, *47*(2), 205-241.

Embrechts, P., McNeil, A., & Straumann, D. (2002). Correlation and dependence in risk management: properties and pitfalls. *Risk management: value at risk and beyond*, *1*, 176-223.

Furman, E., Hackmann, D., & Kuznetsov, A. (2020). On log-normal convolutions: An analytical–numerical method with applications to economic capital determination. *Insurance: Mathematics and Economics*, *90*, 120-134.

Goovaerts, M. J., Kaas, R., & Laeven, R. J. (2010). Decision principles derived from risk measures. *Insurance: Mathematics and Economics*, *47*(3), 294-302.

Grundke, P. (2010). Top-down approaches for integrated risk management: How accurate are they? *European Journal of Operational Research*, *203*(3), 662-672.

Guharay, S., Chang, K. C., & Xu, J. (2018, July). Estimation of Value-at-Risk Using Mixture Copula Model for Heavy-Tailed Operational Risk Losses in Financial, Insurance & Climatological Data. In *2018 21st International Conference on Information Fusion (FUSION)* (pp. 2330-2337). IEEE.

Hasnaoui, H. (2018). Value-at-Risk models for KSA insurance markets: conventional and Takaful. *Academy of Accounting and Financial Studies Journal*, *22*(4), 1-8.

Haugh, M. (2016). *An introduction to copulas. IEOR E4602: quantitative risk management. Lecture notes.* Columbia University.

Huang, J. J., Lee, K. J., Liang, H., & Lin, W. F. (2009). Estimating value at risk of portfolio by conditional copula-GARCH method. *Insurance: Mathematics and economics*, *45*(3), 315-324.

Huang, L., & Shemyakin, A. (2020). Empirical comparison of skewed t-copula models for insurance and financial data. *Model Assisted Statistics and Applications*, *15*(4), 351-361.

Kretzschmar, G., McNeil, A. J., & Kirchner, A. (2010). Integrated models of capital adequacy–Why banks are undercapitalised. *Journal of Banking & Finance*, *34*(12), 2838-2850.

Leković, V. (2018). System of governance in insurance companies according to directive'Solvency II'. *Strani pravni život*, (1), 137-152.

Li, J., Zhu, X., Lee, C. F., Wu, D., Feng, J., & Shi, Y. (2015). On the aggregation of credit, market, and operational risks. *Review of Quantitative Finance and Accounting*, *44*(1), 161-189.

Liang, C., Zhu, X., Li, Y., Sun, X., Chen, J., & Li, J. (2013). Integrating credit and market risk: a factor copula-based method. *Procedia Computer Science*, *17*, 656-663.

Lin, E. M., Sun, E. W., & Yu, M. T. (2018). Systemic risk, financial markets, and performance of financial institutions. *Annals of Operations Research*, *262*(2), 579-603.

Marti, G., Nielsen, F., Bińkowski, M., Donnat, P. (2021). A Review of Two Decades of Correlations, Hierarchies, Networks and Clustering in Financial Markets. In: F. Nielsen (eds), *Progress in Information Geometry*. Signals and Communication Technology (pp. 245–274). Cham: Springer.

Mejdoub, H., & Arab, M. B. (2017). A multivariate analysis for risk capital estimation in insurance industry: Vine Copulas. *Asian Development Policy Review*, *5*(2), 100-119.

Mukhtarov, S., Schoute, M., & Wielhouwer, J. L. (2022). The information content of the Solvency II ratio relative to earnings. *Journal of Risk and Insurance*, *89*(1), 237-266.

Nguyen, T., & Molinari, R. D. (2011). Risk aggregation by using copulas in internal models. *Journal of Mathematical Finance*, *1*(03), 50.

Omari, C. O., Mwita, P. N., & Gichuhi, A. W. (2018). Currency Portfolio Risk Measurement with Generalized Autoregressive Conditional Heteroscedastic-Extreme Value Theory-Copula Model. *Journal of Mathematical Finance*, *8*, 457-477.

Puccetti, G. (2019). Heavy Tails and Copulas. Topics in Dependence Modelling in Economics and Finance. *Quantitative Finance*, *19*(1), 13-14.

Punzo, A., Bagnato, L., & Maruotti, A. (2018). Compound unimodal distributions for insurance losses. *Insurance: Mathematics and Economics*, *81*, 95-107.

Rosenberg, J. V., & Schuermann, T. (2006). A general approach to integrated risk management with skewed, fat-tailed risks. *Journal of Financial economics*, *79*(3), 569-614.

Shim, J. & Lee, S. (2017). Dependency between Risks and the Insurer's Economic Capital: A Copula-based GARCH Model. *Asia-Pacific Journal of Risk and Insurance, 11*(1), 1-29.

Sklar, M. (1959). Fonctions de repartition an dimensions et leurs marges. *Publ. inst. statist. univ. Paris*, *8*, 229-231.

Skoglund, J. (2010). Risk aggregation and economic capital. Retrieved from https://ssrn.com/abstract=2070695.

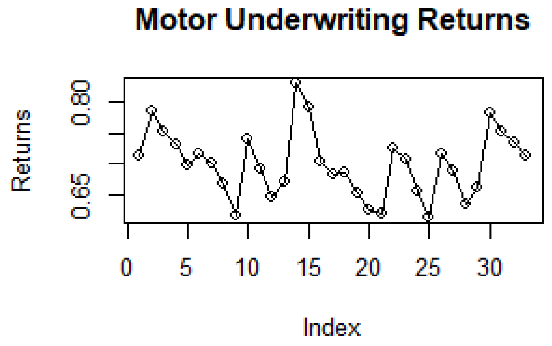
Slepov, V. A., Kosov, M. E., Chalova, A. Y., Gromova, E. I., & Voronkova, E. K. (2019). Integration of the financial market sectors: Factors, risks and management approaches. *International Journal of Mechanical Engineering and Technology*, *10*(2), 1243-1250.

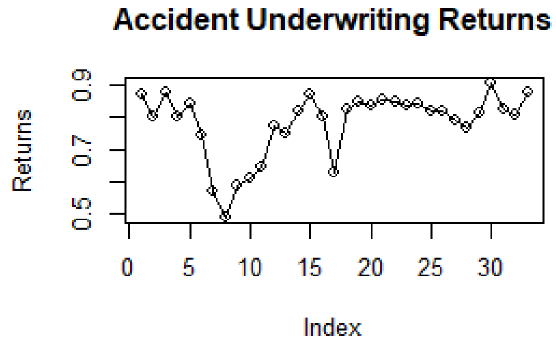
Tang, A., & Valdez, E. A. (2009). Economic capital and the aggregation of risks using copulas. Retrieved from https://ssrn.com/abstract=1347675.

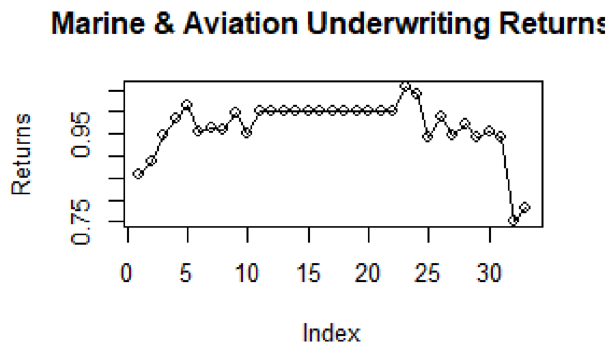
Yan, W., Zhengyin, W., & Yuna, M. (2019). Research on TailVaR-Based Measurement of Economic Capital of Chinese Insurance Companies. *IETI Transactions on Social Sciences and Humanities*, *5*, 112-124.

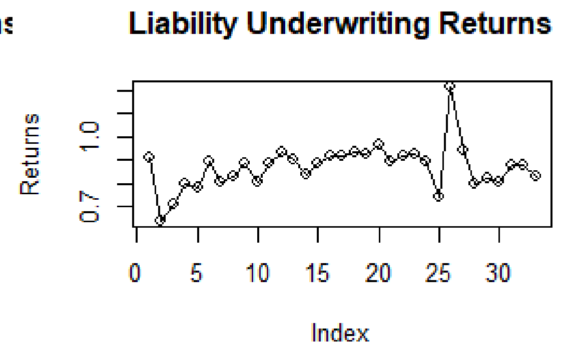
Zeevi, A., & Mashal, R. (2002). Beyond correlation: Extreme co-movements between financial assets. Retrieved from https://ssrn.com/abstract=317122.

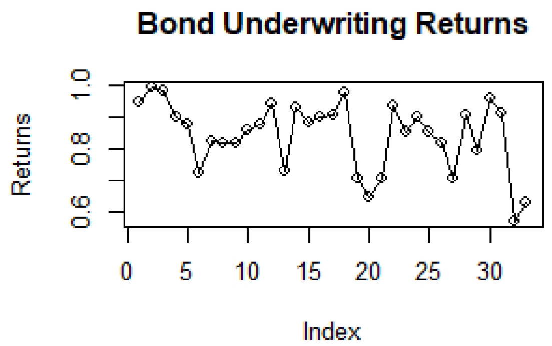
## Appendix

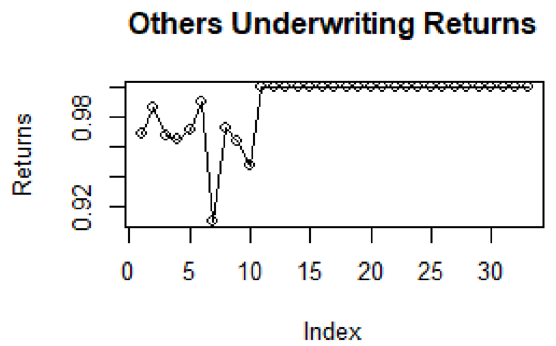

Motor Underwriting Returns
Returns
0.65
0.80
0
5
10
15
20
25
30
Index

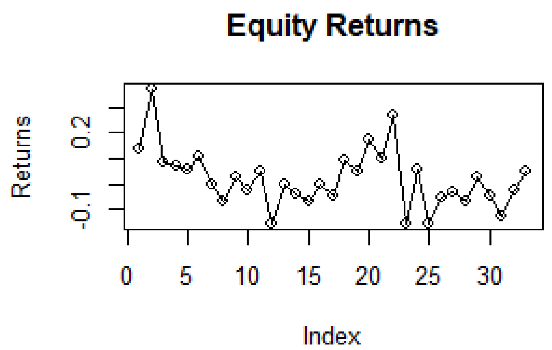

Accident Underwriting Returns
Returns
0.5
0.7
0.9
0
5
10
15
20
25
30
Index

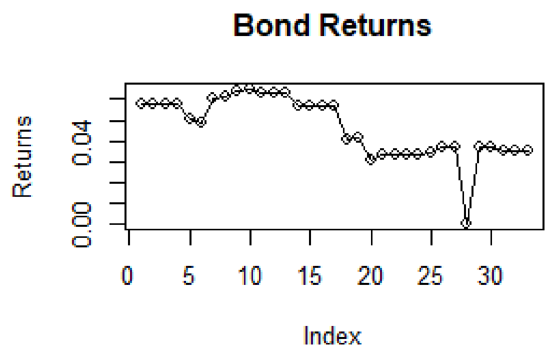

Marine & Aviation Underwriting Returns
Returns
0.75
0.95
0
5
10
15
20
25
30
Index


Liability Underwriting Returns
Returns
0.7
1.0
0
5
10
15
20
25
30
Index

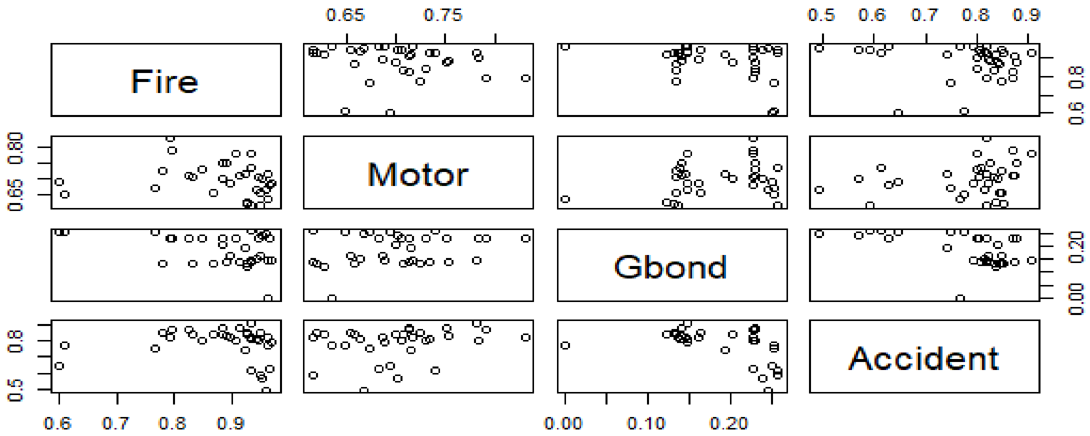

Bond Underwriting Returns
Returns
0.6
0.8
1.0
0
5
10
15
20
25
30
Index


Others Underwriting Returns
Returns
0.92
0.98
0
5
10
15
20
25
30
Index

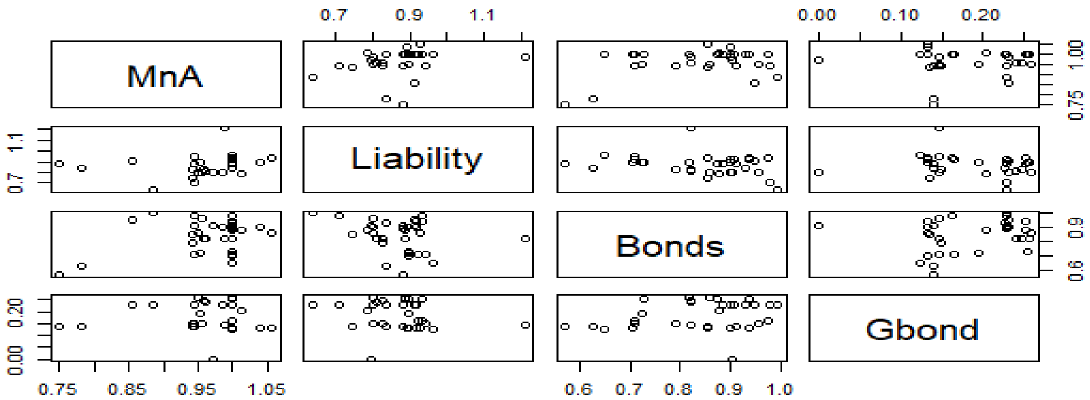

Equity Returns
Returns
-0.1
0.2
0
5
10
15
20
25
30
Index


Bond Returns
Returns
0.00
0.04
0
5
10
15
20
25
30
Index

**Fire, Motor, Accident and GBond Scatterplot**




**MnA, Liability, Bonds and GBond Scatterplot**




**MnA, Liability, Bonds and Stock Scatterplot**