Exploring the Behaviour and Modelling the Timeseries of Six (6) Securities

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Abstract

This study explores the behaviour and models the time series of six selected securities in Nigeria, comprising five sectoral indices and the Nigerian exchange rate (USDNGN), using daily data from the Nigerian Exchange Limited (NGX). The objective is to evaluate the empirical characteristics of these financial time series and identify appropriate models for capturing their volatility dynamics. To achieve this, a suite of volatility models, including ARCH, GARCH, EGARCH, TGARCH, GARCH-M, and PARCH, were applied to the dataset. Model performance was assessed using widely recognized statistical criteria, including Akaike's Information Criterion (AIC), Schwarz's Bayesian Information Criterion (SBIC), Hannan-Quinn Information Criterion (HQIC), and Log-Likelihood values. The models were evaluated for their ability to capture key features such as volatility clustering, asymmetry, and persistence in returns. The results reveal that the logistic distribution provides a better fit for the return distributions of the examined securities compared to traditional normal or lognormal assumptions, due to its ability to account for heavy tails and skewness. Furthermore, among the volatility models, those incorporating asymmetry and power effects, particularly PARCH, demonstrated superior performance, highlighting the importance of model selection in financial time series analysis. This research contributes to the growing literature advocating for more flexible distributional assumptions and advanced volatility models in financial modelling. The findings offer valuable insights for investors, financial analysts, and policymakers seeking to enhance risk assessment and forecasting accuracy in emerging markets like Nigeria.

Keyword: Timeseries, Sectoral Indices, Distribution Fitting, Volatility Models, Performance Criteria

1 Introduction

Financial time series analysis plays a critical role in understanding asset price dynamics, which is vital for informed decision-making by investors and policymakers. These series span various asset classes, each with distinct characteristics and volatility patterns. While the log-normal and normal distributions have traditionally been used to model asset prices and returns, empirical evidence increasingly shows their limitations, especially in capturing extreme values and fat tails (Levy & Levy, 2024). As a result, alternative distributions like the logistic and generalized logistic distributions have gained prominence for their ability to better model the skewness and heavy tails observed in real market data (Gray & French, 2008; Nidhin & Chandran, 2013; Ahmad, 2018). This growing body of research underscores the need for more flexible distributional frameworks that accurately reflect the empirical features of financial markets.

To address limitations in capturing volatility clustering and leptokurtosis in financial returns, Engle (1982) introduced the ARCH model, which was later extended by Bollerslev (1986) through the development of GARCH. These models revolutionized financial econometrics by allowing for time-varying volatility. The GARCH family, including EGARCH (Nelson, 1991) and TGARCH models, offers robust frameworks for understanding persistent and asymmetric volatility patterns. These models have been validated across a wide range of financial markets. For instance, Marisetty (2024) found GARCH (1,1) effective in modelling the volatility of major global indices, while Agunobi, Pam, and Dauda (2024) demonstrated that volatility dynamics differ significantly between developed (UK) and emerging (Nigerian) markets.

Accurately modelling the time series of financial securities is crucial for risk management, portfolio optimization, and strategic planning. However, traditional models often struggle to capture the intricate patterns observed in financial data, such as volatility clustering and leverage effects. This limitation can lead to suboptimal forecasting and increased financial risk. Moreover, the rapid evolution of financial markets, driven by technological advancements and globalization, introduces additional complexities in modelling financial time series. The integration of alternative data sources and the application of advanced machine learning techniques present both opportunities and challenges in enhancing model accuracy and reliability. Therefore, there is a pressing need to investigate and develop models that can effectively capture the behaviour of financial securities, considering the multifaceted nature of modern financial markets.

This paper seeks to investigate the behaviour and model the time series of six securities in Nigeria, thus contributing to the body of literature on time series modelling and the behaviour of Nigeria securities. By providing a reliable predictive tool, this research aims to enhance decision-making for investors, financial managers, and policymakers in Nigeria, ensuring that economic planning is more robust and informed. The remainder of the paper is as follows: Section 2 presents a review of relevant literature, encompassing both theoretical and empirical reviews. Section 3 outlines the data, estimation techniques, and evaluation criteria. Section 4 discusses the results and provides detailed analysis, and Section 5 concludes the study.

2 Review of Relevant Literature

2.1 Empirical Review

Empirical analyses of financial indices often reveal that their price and return distributions deviate from traditional assumptions. Regarding index prices, studies have found that the log-normal distribution does not always provide an adequate fit. For example, a study examining the S&P 500 Index over the period from 1950 to 2005 concluded that the log-normal distribution poorly fits single-period continuously compounded returns, suggesting that future prices may not follow a log-normal distribution (Levy & Levy, 2024). Similarly, when assessing the returns of financial indices, the logistic distribution has been identified as a more suitable model compared to the normal distribution. This is attributed to the logistic distribution's heavier tails, which better capture the extreme values observed in financial returns. For instance, research has demonstrated that the logistic distribution provides a better fit for empirical option prices than the Black-Scholes model, which assumes log-normal returns (Gray & French, 2008; Nidhin & Chandran, 2013).

Furthermore, the generalized logistic distribution has been recognized for effectively capturing the fat-tailed nature of extreme financial returns. In studies involving indices such as the Nikkei 225, this distribution outperformed traditional models like the normal and log-normal distributions (Ahmad, 2018). Concurrently, researchers have examined the performance of various GARCH-type models for modelling volatility dynamics. Marisetty (2024) analysed five global indices and found that GARCH (1,1), EGARCH, and TGARCH models successfully captured market fluctuations, especially during economic shocks like the COVID-19 pandemic. Setiawan et al. (2020) further emphasized the importance of capturing asymmetric volatility using models like APARCH, which outperformed other specifications in predictive accuracy.

Several studies focusing on emerging markets have reinforced the effectiveness of GARCH-type models in capturing persistent volatility. In the Nigerian stock market, Ekong and Onye (2017) showed that symmetric and asymmetric GARCH models provided high predictive accuracy, particularly when using the Generalized Error Distribution (GED). Kuhe (2018) highlighted that including structural breaks and exogenous shocks in the models reduced volatility persistence and improved forecasting performance. Nugroho et al. (2019) and Gyamerah (2019) extended this analysis to both traditional and cryptocurrency markets, confirming the superior performance of models like GJR-GARCH and tGARCH in capturing asymmetries and heavy-tailed behaviour in highly volatile environments.

Other researchers have applied these models across various financial contexts. Mandych et al. (2024) found that GJR-GARCH was particularly effective in modelling credit market volatility in Ukraine, especially during crises. Liu and Hung (2010) observed that GJR-GARCH and EGARCH provided superior forecasts for the S&P 100 index. Tanasya et al. (2020) validated the robustness of asymmetric GARCH models in estimating Value at Risk (VaR) and Expected Shortfall (ES), while Tripathy and Garg (2013) confirmed the prevalence of leverage effects in six emerging markets. Studies by Roy and Shijin (2019) and Lee (2009) further emphasized the relationship between market maturity, policy interventions, and the effectiveness of different GARCH variants.

The empirical studies reviewed provide strong evidence that GARCH-type models remain indispensable for financial market volatility forecasting, with variations in performance depending on asset classes, economic conditions, and structural characteristics. While GJR-GARCH and EGARCH models are widely preferred for capturing asymmetric volatility patterns, recent studies suggest that integrating exogenous macroeconomic indicators, sentiment analysis, and high-frequency trading data can enhance predictive accuracy. Furthermore, advances in hybrid modelling techniques, such as combining GARCH with deep learning methods, provide new opportunities for improving financial risk estimation.

2.2 Theoretical Review

2.2.1 Efficient Market Hypothesis (EMH)

The Efficient Market Hypothesis (EMH), first introduced by Eugene Fama in 1970, posits that financial markets are informationally efficient, meaning that asset prices reflect all available information at any given time. As a result, no investor can consistently achieve returns above the market average, as all information is already incorporated into current prices. Fama (1991) elaborated on EMH by categorizing market efficiency into three forms: weak, semi-strong, and strong. In the weak form, stock prices reflect all historical price data, implying that technical analysis cannot provide any predictive advantage. The semi-strong form posits that prices adjust to all publicly available information, such as earnings reports and news, meaning that neither fundamental nor technical analysis can consistently outperform the market. The strong form suggests that stock prices incorporate even private, insider information, thereby ruling out any opportunity for insider trading to generate excess returns. While EMH has been a central tenet of finance, criticisms have emerged from behavioural finance, which challenges the assumption that all market participants behave rationally. For instance, the introduction of behavioural finance has highlighted how cognitive biases and emotions can lead to market inefficiencies (Malkiel, 2003).

Recent empirical research continues to explore the validity of EMH in different market contexts. Malkiel (2003) critically examined the efficient market hypothesis and its critiques, defending the idea that markets are generally efficient over the long term, despite anomalies that occasionally arise. He emphasized that while anomalies like the January effect or overreactions to news may appear, they do not undermine the overall efficiency of the market in the long run. Similarly, Woo et al. (2020) reviewed global market anomalies and pointed out that while EMH remains a useful framework for understanding financial markets, certain phenomena, such as the day-of-the-week effect, contradict its assumptions. Rossi and Gunardi (2018) investigated stock market anomalies in four European countries, revealing mixed results. While some anomalies were observed in specific countries, their inconsistency across time and markets cast doubt on the universal applicability of EMH. These findings highlight that, while markets tend toward efficiency, investor behaviour and market-specific factors can lead to deviations from perfect efficiency, suggesting that the full acceptance of EMH may need reconsideration in light of new market dynamics and behavioural insights.

2.2.2 Random Walk Theory

The Random Walk Theory, initially proposed by Bachelier (1900) and later popularized by Burton Malkiel (1973), suggests that stock price movements are inherently unpredictable and follow a random path. According to this theory, it is impossible to forecast future stock prices based on historical data, as all available information is quickly reflected in the prices. As a result, market participants cannot consistently outperform the market through techniques such as technical analysis or stock selection. Malkiel (1973) famously stated that stock prices follow a "random walk," implying that price changes are independent of past movements, with no discernible pattern that can be exploited for profit.

Empirical studies have tested the validity of the Random Walk Theory across various markets and time periods. Fama (1965) laid the foundation by testing the hypothesis in the U.S. stock markets and concluded that stock prices follow a random walk, supporting the notion of efficient markets where information is instantly incorporated into prices. Recent studies have also contributed to the ongoing debate over the randomness of stock price movements. For example, Fadda (2019) found evidence supporting the random walk hypothesis, indicating that the price changes in these indexes were not predictable based on historical data. These findings reinforce the notion that while stock prices may exhibit random behaviour in the long run, the influence of investor psychology and market sentiment should not be overlooked.

The mathematical formulation of the Random Walk Theory begins by defining the price of a financial asset at time t as P_t. The model is typically expressed as:

$$P_t = P_{t-1} + \epsilon_t \tag{1}$$

Where:

- P_t: Asset price at time t
- P_{t-1}: Asset price at time $t-1$
- ϵ_t: Random error term (innovation or shock) at time t, often assumed to be i.i.d. (independent and identically distributed), with:

$$\epsilon_t \sim N(0, \sigma^2) \tag{2}$$

A more general version includes a drift component μ, representing a constant expected return or trend:

$$P_t = \mu + P_{t-1} + \epsilon_t \tag{3}$$

Here, μ shifts the mean direction of the walk (e.g., a positive average return over time).

Another version is in terms of returns, if R_t denotes the return from $t-1$ to t, then:

$$R_t = P_t - P_{t-1} = \mu + \epsilon_t \tag{4}$$

This implies that returns are white noise, i.e., they have no autocorrelation and are purely random around a mean μ.

2.2.3 Preferred Habitat Theory

The Preferred Habitat Theory, initially introduced by Culbertson (1957) and later expanded by Modigliani and Sutch (1966), has gained significant attention from both central banks and the financial sector (Vayanos & Vila, 2021). This theory explores the behaviour of bond investors, asserting that they have distinct preferences regarding the maturities of bonds they are willing to purchase. Although investors typically favour shorter-term bonds, they may be inclined to invest in longer-term securities if offered a higher yield to compensate for the additional risks associated with such investments.

In contrast to the Market Segmentation Theory, which posits that investors select bonds based purely on yield regardless of maturity, the Preferred Habitat Theory highlights the importance of maturity preferences alongside yield considerations (Melvin & Norrbin, 2017). This perspective suggests that investors' decisions are influenced by their specific investment goals and their risk tolerance, which leads to a more structured approach to bond purchasing. The theory helps understand how investor preferences influence the bond market and the determination of interest rates. By acknowledging the segmented nature of the market, this theory offers valuable insights into the dynamics of bond yield determination, as investors require a premium to invest in long-term debt instruments. Ultimately, the Preferred Habitat Theory provides a nuanced understanding of the complex relationship between maturity preferences and yield in the bond market.

3 Data, Estimation Techniques and Evaluation Criteria

3.1 Data

The dataset comprises six (6) financial time series with daily frequency, all quoted on the Nigerian Exchange Limited (NGX). Five (5) of these are sectoral and market-wide equity indices namely, the All-Share Index (NGSEINDEX), Industrial Index (NGSEINDUS), Banking Index (NGSEBNK10), Insurance Index (NGSEINS10), and Oil & Gas Index (NGSEOILG5), while the sixth series capture the Nigeria–United States exchange rate (USDNGN). The sample spans from January 2, 2021 to December 31, 2024, a period selected for its economic significance, as it encompasses key events such as the post-pandemic recovery, shifts in monetary policy, and geopolitical developments that likely impacted market volatility and investor behaviour. Daily closing prices were sourced from reputable public data providers, including Bloomberg and Investing.com (see https://www.bloomberg.com/quote/ and https://www.investing.com/), with no adjustments made for public holidays or non-trading days. Although the relatively short sample period and reliance on daily frequency present certain limitations such as constraints on long-term inference and omission of lower-frequency cyclical trends, the approach is justified by the need for high-resolution data to capture short-term volatility dynamics, clustering, and asymmetry. These features are especially pronounced in the post-COVID financial environment and are best modelled using daily observations. The return series, r_t, is computed as the natural logarithmic difference of consecutive daily prices, i.e.,

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{5}$$

where P_t denotes the observed index or exchange rate at time t. This transformation yields the continuously compounded daily returns used for volatility modelling and forecast evaluation.

3.2 Distribution Fitting

To identify the most appropriate probability distribution for the dataset, several distributions were considered, including the normal, exponential, logistic, Weibull, and log-normal distributions. The parameters of each distribution were estimated using Maximum Likelihood Estimation (MLE), a widely accepted method for parameter estimation in statistical modelling. Once the parameters were estimated, the cumulative distribution function (CDF) corresponding to each distribution was obtained. The CDF was then compared to the empirical cumulative distribution function (ECDF) derived from the dataset, allowing for an evaluation of how well the distributions fit the data.

To assess the goodness of fit, the Kolmogorov-Smirnov (KS) test was applied. The KS test compares the empirical CDF of the data with the CDF of each candidate distribution. The test statistic, D_n, is calculated as:

$$D_n = \sup_x |F_n(x) - F(x)| \tag{6}$$

Where $F_n(x)$ represents the ECDF of the data, and $F(x)$ denotes the CDF of the candidate distribution. A larger value of D_n indicates a poorer fit between the observed and theoretical distributions.

The KS test returns a p-value, which is used to determine whether the null hypothesis, that the data follows the distribution, can be rejected. A low p-value suggests that the data does not conform to the distribution, while a high p-value supports the hypothesis that the data follows the distribution. The distribution with the smallest KS statistic (or highest p-value) is considered the best-fitting distribution.

3.3 Volatility Models

The study employed ARCH, GARCH-M, TGARCH, EGARCH, and PARCH models for analysis. The inclusion of these models allows for a comprehensive analysis of financial volatility by capturing key features such as volatility clustering (ARCH/GARCH), risk-return trade-offs (GARCH-M), asymmetric effects of shocks (TGARCH/EGARCH), and flexible tail behaviour (PARCH). Together, these variants enable nuanced modelling of persistence, leverage effects, and nonlinearities inherent in financial time series, improving both understanding and forecasting accuracy.

(a) Autoregressive Conditional Heteroskedasticity (ARCH):

ARCH was developed by in the first seminal paper of Engle (1982). Since then, a lot of literature has emerged for the modelling of heteroskedasticity in financial time series. ARCH models are used to model time-varying volatility (conditional heteroskedasticity). They capture periods of high and low volatility in financial time series data, assuming that volatility is time dependent.

$$y_t = \mu + \epsilon_t \tag{7}$$

where,

$$\epsilon_t = \sigma_t z_t \tag{8}$$

y_t: Return at time t

μ: Mean of the process

ϵ_t: Residuals

σ_t: Conditional standard deviation (volatility)

z_t: White noise error term (e.g., normally distributed)

The conditional variance (σ_t^2) is modeled as

$$\sigma_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \alpha_2\epsilon_{t-2}^2 + \cdots + \alpha_q\epsilon_{t-q}^2 \quad (9)$$

(b) Generalized Autoregressive Conditional Heteroskedasticity (GARCH):

In practical applications of the ARCH (p) model, it often turned out that the number of lag p needed was rather large. To cater for this, Bollerslev (1986) introduced the GARCH (p,q) method by including the lag of the conditional variance into the ARCH model.

The GARCH model is an extension of the ARCH model that incorporates both past squared returns (ARCH terms) and past volatilities (GARCH terms) to model time-varying volatility. The model combines the autoregressive component of the ARCH model with a moving average component:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q}\alpha_i\epsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 \quad (10)$$

where,

σ_t^2 is the conditional variance

α_i and β_j are parameters to be estimated.

(c) Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH):

Introduced by Nelson (1991), the EGARCH model address some limitations of GARCH by allowing for asymmetric effects of positive and negative shocks on volatility. This model ensures that the variance remains positive at all times by modeling it in an exponential form.

$$\log(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p}\alpha_i\frac{\epsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q}\beta_j\log(\sigma_{t-j}^2) \quad (11)$$

The logarithmic transformation ensures that the conditional variance (σ_t^2) is always positive.

(d) Threshold Generalized Autoregressive Conditional Heteroskedasticity (TGARCH):

Developed by Engle and Ng (1993), the TGARCH model extends the GARCH model by allowing for different impacts of positive and negative returns on volatility (asymmetric volatility). This model introduces a threshold variable to distinguish the effects of positive and negative shocks.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q}\alpha_i\epsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 + \gamma(\mathbb{I}[\epsilon_{t-1} < 0])\epsilon_{t-1}^2 \quad (12)$$

where,

$\mathbb{I}[\epsilon_{t-1} < 0]$ is an indicator function that takes the value 1 if the past shock is negative (bad news) and 0 if it is positive (good news), and γ represents the asymmetry parameter.

(e) Generalized Autoregressive Conditional Heteroskedasticity in Mean (GARCH-M):

The GARCH-M model is an extension of the GARCH model that allows the conditional variance to affect the mean equation. The model was introduced by Engle, Lilien and Robins (1983), and it is particularly useful in financial models where the risk (volatility) influences the expected return. The model can be expressed as

$$y_t = \mu + \lambda\sigma_t + \epsilon_t \tag{13}$$

where λ is the risk premium, and the conditional variance σ_t^2 follows a GARCH process. The volatility enters the mean equation, allowing risk to affect the expected return.

(f) Power ARCH (PARCH):

The PARCH model generalizes the ARCH model by allowing for the conditional variance to depend on powers of the past squared residuals. This provides greater flexibility to model different degrees of volatility persistence.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i |\epsilon_{t-1}|^p \tag{14}$$

where p is a parameter that controls the degree of persistence in volatility. When p = 2, the PARCH model reduces to the standard ARCH model.

3.4 Performance Criteria

The performance of the fitted model is evaluated using several well-established statistical criteria, namely, Akaike's Information Criterion (AIC), Schwartz's Bayesian Information Criterion (SBIC), Hannan-Quinn Information Criterion (HQIC), and Log-Likelihood. These criteria, defined in equations 15-18 below, are essential for understanding how well the model fits the data while accounting for model complexity (Akaike, 1974; Schwarz, 1978; Hannan & Quinn, 1979; Bollerslev, 1986; Dallah & Adeleke, 2009).

The AIC is a widely used measure for model selection that balances goodness of fit and model complexity. Specifically, it provides a relative ranking of models based on their ability to explain the observed data, with a penalty for excessive complexity. When comparing multiple models, the one with the lowest AIC is considered the best, as it indicates a model that explains the data efficiently without overfitting. The AIC is useful in model selection because it strikes a balance between goodness of fit and parsimony, making it a robust tool for time series analysis and forecasting.

The Schwartz's Bayesian Information Criterion (SBIC), is another widely used criterion in model selection, which imposes a stronger penalty for model complexity compared to AIC, making it more conservative when selecting a model. This characteristic makes the SBIC especially suitable when working with large datasets or when the risk of overfitting is a concern. Like the AIC, the SBIC allows for the comparison of different models, with the model exhibiting the lowest SBIC being considered the best. However, the more severe penalty for additional parameters makes the SBIC preferable when the aim is to identify a more parsimonious model.

The Hannan-Quinn Information Criterion (HQIC), is another model selection criterion that is less sensitive to sample size than the AIC and BIC. Similar to the AIC and SBIC, it penalizes models with excessive complexity, but it does so in a way that is less sensitive to sample size compared to the AIC and SBIC. The HQC provides a reasonable balance between model fit and complexity, offering a valuable alternative when the sample size is small and ensuring that the chosen model is not too complex for the available data.

The Log-Likelihood (LL) is a measure of how well the model explains the observed data. For GARCH models, it is derived from the likelihood of the residuals and their conditional variances. The higher the log-likelihood, the better the model fits the data.

These criteria offer a comprehensive framework for evaluating and comparing different models. Informed decisions are made by selecting the model that not only fits the data well but also avoids overfitting by penalizing unnecessary complexity. Each criterion has its strengths, and when used in combination, they provide a robust methodology for model selection and performance evaluation.

$$AIC = -2\mathcal{L}(\theta) + 2k \tag{15}$$

$$SBIC = -2\mathcal{L}(\theta) + k\ln(n) \tag{16}$$

$$HQIC = -2\mathcal{L}(\theta) + 2k\ln(\ln(n)) \tag{17}$$

$$\mathcal{L}(\theta) = -\frac{n}{2}log(2\pi) - \frac{1}{2}\sum_{t=1}^{n}\log(h_t) - \frac{1}{2}\sum_{t=1}^{n}\frac{e_t^2}{h_t} \tag{18}$$

Where:

$\mathcal{L}(\theta)$ is the log-likelihood;

n the number of observations;

h_t is the conditional variance at time t (calculated from the GARCH model).

e_t is the residual at time t

θ are the parameters of the GARCH model.

4 Results and Discussion

4.1 Results of Fitted Distribution using QuickFit

The dataset for the various index data were fitted to a distribution to ascertain the distribution with best fit. The result of fitted distribution to the price and return of each index is below.

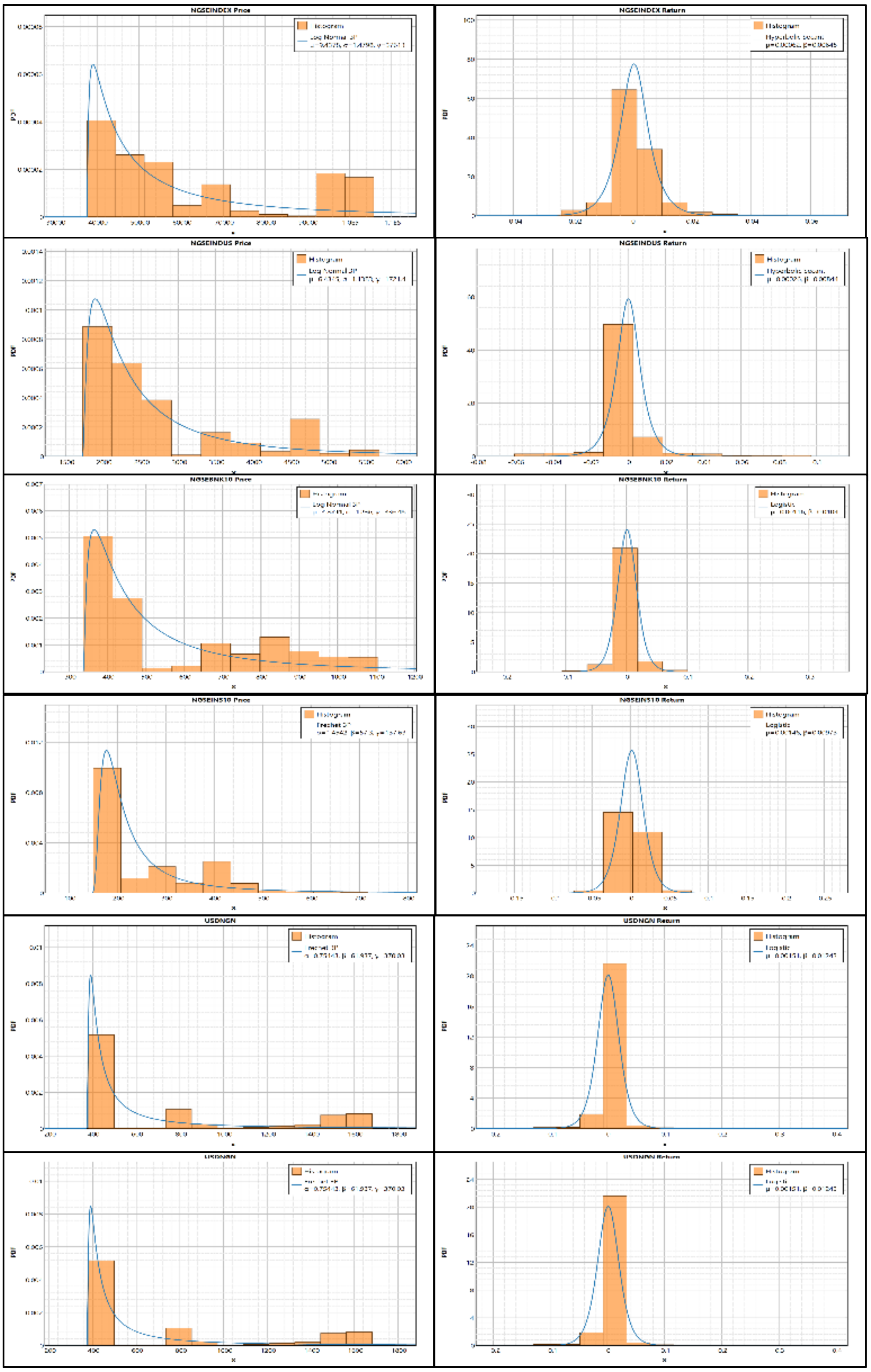


Figure 4.1: Fitted distributions of six securities

For the index price, four of the six indexes followed a lognormal 3P distribution and the remaining two followed a Frechet 3P distribution. In a similar manner, the logistic distribution is the distribution that demonstrates a superior fit for the returns of the indexes, which is in tandem with many literatures that showed that financial returns follow a logistic distribution (see. Levy & Levy, 2024; Ahmad, 2018).

4.2 GARCH Models

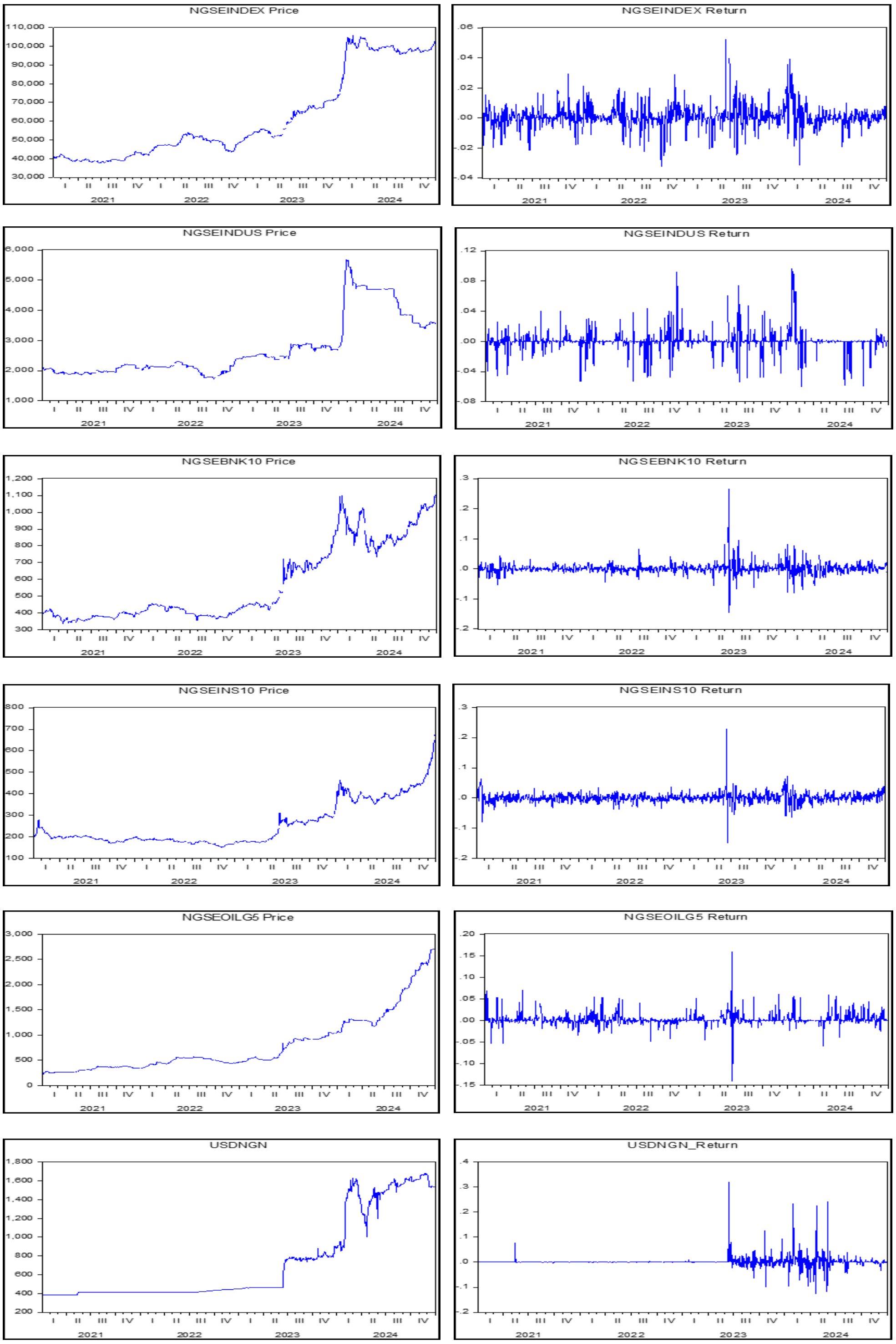
The table below displayed the descriptive statistics of six (6) securities.

Table 4.1: Descriptive Statistics for the return of various securities

Descriptive Statistics	**NGSEINDEX**	**NGSEINDUS**	**NGSEBNK10**	**NGSEINS10**	**NGSEOILG5**	**USDNGN**
Mean	0.000956	0.000615	0.001163	0.001457	0.002623	0.001574
Median	0.000275	0.00000397	0.000552	0.001063	0.000000	0.000000
Maximum	0.052325	0.097166	0.265205	0.229837	0.160146	0.319694
Minimum	-0.032267	-0.060219	-0.146256	-0.148753	-0.14124	-0.12682
Standard Deviation	0.007636	0.014277	0.018866	0.017645	0.014905	0.022502
Skewness	0.665744	1.299363	2.597611	1.632875	1.680092	5.785262
Kurtosis	9.009405	17.254450	48.27561	37.46889	31.2437	78.91453
Jarque-Bera	1561.211	8651.390	85584.39	49399.31	33337.44	255777.7
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

NGSEINDEX is the Nigeria All Share Index, NGSEINDUS is the Nigeria Industrial Index, NGSEBNK10 is the Nigeria Banking Sector Index, NGSEINS10 is the Nigeria Insurance Sector Index, NGSEOILG5 is the Nigeria Oil Sector Index, and USDNGN is the exchange rate for United States Dollar to Nigerian Naira.

Figure 4.1 illustrates a series of two distinct line charts that provide a comprehensive view of the behaviour of each of six securities over time. The first chart displays the price movements of each security, offering a visual representation of how each security's price has evolved throughout the period under consideration. This chart helps to highlight trends, fluctuations, and any notable price changes, allowing for a deeper understanding of the market performance of these assets. The second chart, which is positioned alongside the first, presents the returns of each security. The return chart is particularly useful for assessing the risk and performance of the securities.



NGSEINDEX is the Nigeria All Share Index, NGSEINDUS is the Nigeria Industrial Index, NGSEBNK10 is the Nigeria Banking Sector Index, NGSEINS10 is the Nigeria Insurance Sector Index, NGSEOILG5 is the Nigeria Oil Sector Index, and USDNGN is the exchange rate for United States Dollar to Nigerian Naira.

Figure 4.2: Line chart for six securities

Table 4.2: Stationarity Test for the securities

Bond maturities	**ADF test (No differencing)**	
	Statistic	**p-value**
NGSEINDEX	-25.359190	0.0000
NGSEINDUS	-10.73344	0.0000
NGSEBNK10	-24.34005	0.0000
NGSEINS10	-21.48712	0.0000
NGSEOILG5	-27.63416	0.0000
USDNGN	-21.15699	0.0000

Source: Author's computation

To test for stationarity, the Augmented Dickey-Fuller (ADF) test was done. For all the securities, the ADF test results strongly suggest that the null hypothesis of a unit root can be rejected at conventional significance levels (e.g., 1%, 5%, or 10%), as evidenced by the highly negative t-statistics and p-values of 0.0000. This indicates that the returns of these securities are stationary without requiring differencing, and they are well-suited for modelling and forecasting, as their statistical properties do not change over time.

Table 4.3: Test of Heteroskedasticity

ARCH Effects	**NGSEINDEX**	**NGSEINDUS**	**NGSEBNK10**	**NGSEINS10**	**NGSEOILG5**	**USDNGN**
F-statistic	24.46874	151.6077	45.09981	15.42005	104.0162	10.84662
p-value	0.0000	0.0000	0.0000	0.0001	0.0000	0.0010

Source: Author's computation

The p-value from table 4.3 are statistically significant. This suggests the presence of ARCH effects**,** meaning the variance of the residuals is time-varying, conditional on past shocks, and exhibits volatility clustering, which implies that the securities may require a model like GARCH to account for their time-varying volatility.

Table 4.4 reports model performance across multiple financial indices and the exchange rate (USD/NGN), using standard model selection criteria such as Akaike Information Criterion (AIC), Schwarz Bayesian Information Criterion (SBIC), Hannan-Quinn Information Criterion (HQIC), and log-likelihood (LL). The analysis reveals that no single GARCH-type model dominates across all datasets, underscoring the importance of model selection in relation to the distinct volatility dynamics embedded within each time series. Across all datasets, the basic ARCH model consistently exhibits higher AIC, SBIC, and HQIC values compared to its extensions, indicating that GARCH-type models provide a superior fit. For the NGSEINDEX dataset, the PARCH model exhibits the lowest AIC (-7.107352), values, along with the highest log-likelihood (3518.032), suggesting it offers the best performance among the evaluated models.

Table 4.4: Performance Metrics of Fitted Models for various Securities

Data	Models	Statistic			
		AIC	SBIC	HQIC	LL
NGSEINDEX	ARCH	-7.026854	-7.007034	-7.019317	3475.266
	GARCH	-7.105678	-7.080903	-7.096256	3515.205
	EGARCH	-7.095913	-7.066182	-7.084606	3511.381
	TGARCH	-7.106911	-7.07718	-7.095604	3516.814
	GARCH-M	-7.106847	-7.077116	-7.09554	3516.782
	PARCH	-7.107352	-7.072666	-7.094161	3518.032
NGSEINDUS	ARCH	-5.924672	-5.904851	-5.917134	2930.788
	GARCH	-5.958745	-5.933969	-5.949323	2948.62
	EGARCH	-5.954769	-5.925038	-5.943462	2947.656
	TGARCH	-5.963548	-5.933817	-5.952242	2951.993
	GARCH-M	-5.96619	-5.93646	-5.954884	2953.298
	PARCH	-5.962053	-5.927367	-5.948862	2952.254
NGSEBNK10	ARCH	-5.595695	-5.575875	-5.588158	2768.273
	GARCH	-5.731072	-5.706296	-5.72165	2836.15
	EGARCH	-5.722356	-5.692626	-5.71105	2832.844
	TGARCH	-5.730979	-5.701248	-5.719672	2837.104
	GARCH-M	-5.729611	-5.69988	-5.718304	2836.428
	PARCH	-5.732487	-5.697801	-5.719296	2838.848
NGSEINS10	ARCH	-5.592894	-5.573073	-5.585356	2766.89
	GARCH	-5.712662	-5.687886	-5.70324	2827.055
	EGARCH	-5.729093	-5.699363	-5.717787	2836.172
	TGARCH	-5.733759	-5.704029	-5.722453	2838.477
	GARCH-M	-5.711189	-5.681458	-5.699883	2827.327
	PARCH	-5.735537	-5.700851	-5.722346	2840.355
NGSEOILG5	ARCH	-5.728311	-5.70849	-5.720773	2833.786
	GARCH	-5.755965	-5.73119	-5.746543	2848.447
	EGARCH	-5.840282	-5.810551	-5.828975	2891.099
	TGARCH	-5.801805	-5.772074	-5.790499	2872.092
	GARCH-M	-5.754108	-5.724377	-5.742802	2848.529
	PARCH	-5.840866	-5.80618	-5.827675	2892.388
USDNGN	ARCH	-5.056569	-5.037543	-5.049352	2633.416
	GARCH	-5.265437	-5.241654	-5.256415	2743.027
	EGARCH	-5.509764	-5.481224	-5.498937	2871.077
	TGARCH	-5.304203	-5.275663	-5.293376	2764.186
	GARCH-M	-5.271936	-5.243395	-5.261109	2747.407
	PARCH	-5.310698	-5.277401	-5.298067	2768.563

Source: Author's computation
AIC is Akaike Information Criterion, SBIC is Schwarz Bayesian Information Criterion, HQC is Hannan-Quinn Information Criterion, and LL is Log-Likelihood.

A similar trend is observed in the NGSEINDUS dataset, where the GARCH-M model outperforms others with the lowest AIC (-5.96619), SBIC (-5.93646), HQIC (-5.954884), and highest log-likelihood (2953.298). In the NGSEBNK10 and NGSEINS10 datasets, the PARCH model consistently exhibits the best performance, as indicated by the lowest AIC and highest LL values (2838.848 and 2840.355, respectively). This suggests that PARCH may better capture the volatility structure in these financial series. For NGSEOILG5, both the PARCH and EGARCH models perform well, with PARCH displaying the lowest AIC (-5.840866) and the highest LL (2892.388), closely followed by EGARCH. This suggests that asymmetric effects may be present in oil-related financial data. Finally, for the USDNGN exchange rate data, the EGARCH model demonstrates superior performance, as reflected in the lowest AIC (-5.509764) and the highest LL (2871.077). This result aligns with the understanding that exchange rate volatility often exhibits asymmetric properties, which EGARCH models are designed to capture.

The Power ARCH (PARCH) model demonstrates superior performance for the banking and insurance sector indices. This result is theoretically consistent with the nature of financial return series in these sectors, which often exhibit volatility clustering, long memory, and leverage effects. Unlike standard GARCH models, the PARCH framework introduces a flexible power term (δ) that allows the conditional standard deviation to respond nonlinearly to past innovations. This is particularly valuable in capturing persistent and asymmetric volatility patterns typical in highly regulated and macro-sensitive sectors like banking and insurance. These sectors are also exposed to systemic risk and policy interventions, which can lead to sharp volatility swings that are better captured through the additional flexibility embedded in the PARCH structure. The superior performance of the Exponential GARCH (EGARCH) model in modelling the USD/NGN exchange rate is consistent with the known features of exchange rate dynamics in emerging markets. Exchange rate volatility, particularly in economies such as Nigeria's, is often driven by external shocks, speculative pressures, political risk, and macroeconomic asymmetries, factors that lead to nonlinear and asymmetric responses in volatility. EGARCH explicitly models the log of conditional variance and allows for asymmetric effects, whereby positive and negative shocks of equal magnitude have differing impacts on volatility. The USD/NGN exchange rate has historically exhibited higher sensitivity to negative news (e.g., oil price declines, devaluation fears, policy uncertainty) than to positive developments. The EGARCH model, by design, captures such effects through its leverage term, which accounts for the sign and magnitude of past shocks. Thus, its superior performance is not only statistically significant but also econometrically appropriate given the stylized facts of exchange rate behaviour in developing economies.

Table 4.5: Prediction performance for models obtained for various Securities

		Statistic			
Data	**Models**	**AIC**	**SBIC**	**HQIC**	**LL**
NGSEINDEX	PARCH	-7.107352	-7.072666	-7.094161	3518.032
NGSEINDUS	GARCH-M	-5.96619	-5.93646	-5.954884	2953.298
NGSEBNK10	PARCH	-5.732487	-5.697801	-5.719296	2838.848
NGSEINS10	PARCH	-5.735537	-5.700851	-5.722346	2840.355
NGSEOILG5	PARCH	-5.840866	-5.80618	-5.827675	2892.388
USDNGN	EGARCH	-5.509764	-5.481224	-5.498937	2871.077

Source: Author's computation

The results indicate that more flexible GARCH-type models, particularly PARCH and EGARCH, tend to outperform the basic ARCH model across various financial time series. These findings emphasize the importance of incorporating asymmetric and power effects in modelling financial market volatility. The comparative analysis of ARCH and its GARCH extensions across various datasets aligns with existing literature emphasizing the efficacy of GARCH-type models in capturing financial time series volatility. For instance, Nugroho et al. (2019) conducted an empirical study comparing GARCH, GARCH-M, GJR-GARCH, and log-GARCH models using daily data from indices such as the DJIA and S&P 500, concluding that the GJR-GARCH model demonstrated the best overall fit to the data. Similarly, Ekong and Onye (2017) investigated the Nigerian stock market and found that GARCH(1,1) and augmented EGARCH(1,1) models, particularly under the Generalized Error Distribution (GED), exhibited superior forecasting capabilities. However, in a study by Setiawan et al. (2020), the study concluded that the Asymmetric Power GARCH (APARCH) model outperformed other asymmetric models, including EGARCH and TGARCH, in modelling daily stock return volatility, as evidenced by lower AIC and SBIC values. These findings corroborate the current results, where models like PARCH and EGARCH demonstrated superior performance metrics, underscoring the importance of accommodating asymmetries in financial volatility modelling.

5 Conclusion

This study set out to examine the behaviour and model the time series of six securities in Nigeria using daily data from the NGX. Volatility models including ARCH, GARCH, EGARCH, TGARCH, GARCH-M, and PARCH were applied, with performance assessed using AIC, SBIC, HQIC, and Log-Likelihood criteria. The analytical process involved fitting and comparing these models to evaluate their effectiveness in capturing the dynamics of each time series. The analysis of financial returns reveals that the logistic distribution offers a more accurate representation of the returns of the examined indexes, aligning with findings from previous studies (e.g., Levy & Levy, 2024) that highlight its effectiveness in capturing the empirical characteristics of financial market behaviour. The heavy tails and skewness observed in the returns suggest that the logistic distribution is better suited to model extreme values and fluctuations in the market compared to more traditional distributions like the normal or lognormal. This reinforces the growing recognition in the literature that the logistic distribution, with its ability to account for the non-normal features of financial returns, offers a more accurate and reliable framework for understanding and forecasting financial market dynamics. Consequently, this study supports the need for adopting flexible distributional models, such as the logistic distribution, to enhance the precision of financial modelling and risk assessment.

The comprehensive analysis of ARCH and its various GARCH extensions across multiple financial datasets underscores the critical importance of selecting appropriate volatility models to accurately capture the inherent dynamics of financial time series. The findings indicate that models incorporating asymmetry and power effects, such as the PARCH and EGARCH models, often outperform the standard ARCH model. This is evidenced by their superior performance metrics, including lower values of the Akaike Information Criterion (AIC), Schwarz Bayesian Information Criterion (SBIC), and Hannan-Quinn Information

Criterion (HQIC), as well as higher log-likelihood (LL) values. These model-specific insights provide practical guidance for portfolio managers in selecting volatility forecasting tools tailored to different asset classes and sectors, thereby supporting more informed investment decisions and risk-adjusted asset allocation strategies.

These results are consistent with existing literature that emphasizes the efficacy of asymmetric GARCH models in capturing financial time series volatility. For instance, Nugroho et al. (2019) conducted an empirical study comparing GARCH, GARCH-M, GJR-GARCH, and log-GARCH models using daily data from indices such as the DJIA and S&P 500. Their findings indicated that the GJR-GARCH model yielded the most satisfactory fit, highlighting the significance of accounting for asymmetries in volatility modelling. Similarly, Ekong and Onye (2017) investigated the Nigerian stock market and found that GARCH(1,1) and augmented EGARCH(1,1) models, particularly under the Generalized Error Distribution (GED), exhibited superior forecasting capabilities. This underscores the effectiveness of models that capture asymmetries and leverage effects in financial time series.

Furthermore, Gyamerah (2019) examined the volatility of Bitcoin using various GARCH-type models and found that the TGARCH model, which accounts for asymmetries, was the most effective in capturing the unique characteristics of Bitcoin returns. This underscores the importance of considering asymmetric models when dealing with financial assets that exhibit non-linear behaviours. The superior performance of asymmetric GARCH models can be attributed to their ability to capture leverage effects, where negative shocks have a more pronounced impact on volatility than positive shocks of the same magnitude. This characteristic is crucial for accurate risk assessment and forecasting in financial markets. From a policy standpoint, the sector-specific volatility patterns revealed by the preferred models offer regulators empirical tools for monitoring financial stability, designing targeted interventions, and anticipating systemic risk transmission.

The findings from this analysis align with the broader consensus in financial econometrics that incorporating asymmetries and non-linearities leads to more robust and reliable volatility modelling. The empirical evidence from this study and supporting literature highlights the necessity of employing advanced GARCH-type models that account for asymmetries and power effects to effectively model and forecast volatility in financial time series. This approach enhances the accuracy of risk management strategies and investment decisions in both developed and emerging financial markets.

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