SAFETY-FIRST PORTFOLIO OPTIMIZATION MODEL: SIMULATING THE ASSET PORTFOLIO OF CHINESE INSURANCE FUNDS WITH DIRECT INVESTMENT IN STOCK MARKET

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Abstract

*The Safety-First Portfolio Optimization Model (S-F Model) was first discovered by Roy in 1952, further developed by followers and highly valued by Markowitz in 1999. We used the S-F Model and the traditional Mean-Variance Portfolio Optimization Model (M-V Model) for a simulation analysis on China insurance fund investment into the capital market of Mainland China and Hong Kong, respectively. We found that, analyzed by the S-F model, the proportions of investment into Hong Kong capital markets in all of the optimal investment portfolios are always less than that predicted by the M-V Model. Based upon the analysis of these different simulation results, we extended the hypothesis boundaries of both S-F Model and M-V Model to the reality of today's fast-developing global capital markets, especially the possibility of the portfolio loss associated with infrequent catastrophic events. Thus finally, we will present a general framework of Safety-First Mean-higher-Moments Portfolio Optimization Model (SFMM Model), which is of great significance for China insurance funds to make full use of overseas capital markets to diversify the domestic systematic risks so as to increase its return of investment. The theoretical analysis of this research will offer suggestions to the China insurance industry about the optimal capital structure after direct investment into China mainland stock market and Hong Kong stock market. The research outcome will provide important references for China financial authorities and insurance industry for their relevant decision-*making.

Keywords: China Insurance Fund; Capital Market; Safety-First; Extreme-Value; Risk-Bearing;

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Portfolio Optimization; SFMM Model

Introduction

In the same year of 1952 when Markowitz's paper "Portfolio Selection: Efficient Diversification of Investments" was published on the *Journal of Finance*, A. D. Roy's paper "Safety-First and the Holding of Assets" appeared on the *Econometrics*, introducing the concept of safety-first for the first time. Roy took a consideration that a dreadful event might substantially erode an investor's wealth, making safety-first germane to questions about portfolio optimization. Safety-first considers the investor's desire to minimize the probability of large negative returns.

It was in 1955 that Kataoka (Elton, 1955)[1] modified Roy's approach by pre-specifying the acceptance chance of an undesirable outcome, and then, selecting the portfolio with the highest critical return at that probability. Then Telser combined the criteria of both Roy and Kataoko, such that the expected return maximization constrained by a limit to the probability that the return could be less than some pre-specified critical return under the optimal safety-first portfolio (Telser, 1955). However, Roy, Kataoka and Telser failed to order the return rate of risky assets. Thus, Arzac and Bawa studied the portfolio choice when risk aversion investors behaved according to S-F rule, arguing a complete ordering of all risky assets (Arzac, 1977).

It is known that Markowitz's (1952) Mean-Variance (M-V) model is based on the assumptions of investors' quadratic preferences and normal distribution of investment returns.[2] However, these assumptions are deviated greatly from the reality of today's global capital markets. What Markowitz cannot consider is that the return distributions of emerging capital markets differ sharply from normality, with investment weights favoring positive skewness and kurtosis (Mahfuzul Haque, et al, 2004). Pownall and Koedijk (1999) indicated that the financial asset returns are fat-tailed. Longin and Solnik (2001) found that international equity market correlations are greatly affected by market trend in times of extreme returns. Under the situation of a bear market, the trend of extreme return becomes more serious.

S-F Model has not been widely used because of its complicated calculation of the extreme return probability. Jansen (2000) introduced the use of extreme value analysis into the portfolio selection with S-F rule, which showed that the S-F rule would be successfully improved by exploring the fat tail property of asset returns. However, there are rare publications of case studies.

The Chinese financial market is emerging. In order to ensure the security of insurance companies' operation, the financial authorities of China have long confined the investment of Chinese

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[1] P237-240.
[2] (Levy, 1992).

insurance funds merely into bank deposits and government bonds but no permission for equities or corporate bonds. However, such narrow ranged investment tools have already resulted in the low rate of investment return for Chinese insurance funds. There has been strong desire to broaden the investment channels and tools such as to direct invest into stock markets to pursue higher return of insurance funds. In October 2004, the declaration of «Temporary Regulations on Insurance Institutional Investment into Stock Market» symbolized the beginning of the Chinese insurance fund direct investment into stock markets. However, it is obvious that the investment risk in stock market is much higher than that in bank deposits and government bonds. In particular, because of the premature market condition of the Chinese stock market and the special solvency requirement of insurance industry, the maximizing investment return of China insurance funds definitely has to be constrained under the safety-first principal. It is an important and practical issue that the Chinese insurance industry and authorities must deal with properly. In this study, we will use the Safety-First Portfolio Optimization Model to simulate the theoretical optimal asset portfolio structure for China insurance fund after direct investment into stock markets. We will also compare the differences of the simulated results using the Safety-First Portfolio Optimization Model versus the M-V Portfolio Optimization Model. The theoretical analysis of this research will offer suggestion to the China insurance industry about the optimal capital structure and the estimated scale of direct investment into the stock markets after 2005. The research outcome will provide important references for the China financial authorities and insurance industry for their relevant decision-making.

Safety-First Portfolio Optimization Model

Roy's (1952) Safety-First (S-F) rule was developed by the following researchers such as Arzac (et al, 1977), Jansen (et al, 2000) and Mahfuzul Haque (et al, 2004) etc., gradually into a set of theory that is named Safety-First Portfolio Optimization Model in this research. It can be summed up as the following:

For a risk aversion investor, let W_0 denotes his initial wealth, let w denotes his expected wealth net of financing cost, let s denotes his critical level of wealth, let ξ denotes his maximal acceptable probability of a disaster,

If we denote $P = \Pr\{w \le s\} \le \xi$, π=1

P is the happening probability of the event $\{w \le s\}$, that is, the happening probability of the event with which the investor's expected wealth net of financing cost is no more than his critical level of wealth.

Thus we have $P = \Pr\{w \le s\} > \xi$, =1-P

Therefore the preference ordering by (π, w) can describe the investor's risk aversion attitude and risk bearing.

Let us suppose there are one risk-free asset and i kinds of risk assets. Let r denote the risk-free gross rate of return (which is equal to one plus net risk-free rate or return), X_i denotes the initial market value of the *ith* risk asset, Y_i denotes the final market value of *ith* risk asset, V_i denotes the amount of the *ith* risk asset.

A risk aversion investor can borrow or lend on the financial market. Let b denote the amount of his lending (b<0 represent borrowing).

Then $W_0 = \sum_i V_i X_i + b \qquad w = \sum_i V_i Y_i + br$

Mahfuzul, Haqueÿand et al developed the model of risk aversion safety-first investor (Mahfuzul, Haque, et.al., 2004):

$P = \Pr\{(\sum_i V_i Y_i + br) \le s\} \le \xi, \pi = 1; otherwise, \pi = 1 - P$ (given π)

MAX (π, w) (expected wealth maximization)

S. t. $W_0 = \sum_i V_i X_i + b$ (budget constraint) (1)

For safety-first condition:

as $P = \Pr\{(\sum_i V_i Y_i + br) \le s\} \le \xi;$ so $P = \Pr\{\sum_i V_i Y_i \le s - br\} \le \xi$

as $W_0 = \sum_i V_i X_i + b;$ so $\sum_i V_i X_i = W_0 - b$

Thus $\Pr\left(\frac{\sum_i V_i Y_i}{\sum_i V_i X_i} \le \frac{s - br}{W_0 - b}\right) \le \xi$

Let $R = \frac{\sum_i V_i Y_i}{\sum_i V_i X_i}$, $C_\xi(R) = r + \frac{s - W_0 r}{W_0 - b}$

Thus $\Pr\{R \le C_\xi(R)\} \le \xi$ (2)

From the above denote, we can see R implies the gross expected rate of return of all risk assets (which is equal to one plus net rate of return of all risk assets).

The maximum of risk assets that a risk aversion safety-first investor buys must satisfy the above equation.

Thereinto, $C_\xi(R)$ is determined by the ξ and the distribution of portfolio.

$$C_\xi(R) = 1 - \left| the\ lower\ tail\ exceedence\ value \right|^3$$

Therefore, $C_\xi(R)$ can measure the risk that may be preferred to the second moment, as it is based on large negative returns. However, in some cases such as in normal distribution, S-F Model is equivalent to M-V optimization.

For the condition that violates the Safety-First rule

$$C_\xi(R) < r + \frac{s - W_0 r}{W_0 - b} \tag{3}$$

If the investor's critical wealth s is less than the secure final wealth $W_0 r$, then he will decline a fair risk and favor pure lending at the gross risk-free rate r.

Under favorable risk condition

When $C_\xi(R) = r + \frac{s - W_0 r}{W_0 - b}$, then the investor will maximize his wealth (4)

As $C_\xi(R) = r + \frac{s - W_0 r}{W_0 - b}$; then $W_0 - b = \frac{s - W_0 r}{C_\xi(R) - r}$

As $R = \frac{\sum_i V_i Y_i}{\sum_i V_i X_i}$

From $w = \sum_i V_i Y_i + br$; then $w = R\sum_i V_i X_i + br$

From $W_0 = \sum_i V_i X_i + b$; then $w = R(W_0 - b) + br = (W_0 - b)(R - r) + W_0 r$

Thus $w = W_0 r + \frac{s - W_0 r}{C_\xi(R) - r}(R - r)$

Therefore the investor should maximize his wealth:

$$Max \rightarrow w = W_0 r + \frac{s - W_0 r}{C_\xi(R) - r}(R - r)$$

Let $$r_p = \frac{R - r}{r - C_\xi(R)} \tag{5}$$

[3] The detailed analysis on the method of extreme value theory can be found in Reiss (2001).

Thus $\quad Max \rightarrow w = W_0 r - (s - W_0 r) r_p$ (6)

There into r_p is the risk premium.

Thus, the Safety-First portfolio decision involves maximization of r_p to the return opportunity loss $(s - W_0 r)$ that the investor is willing to incur with probability ξ.

The investor also determines the optimal V_i in the portfolio. The investor has a probability $(1-\xi)$ or greater of maintaining a value in excess of his critical wealth s.

The investor can also determine the scale of the risky investment with average.

As $\quad W_0 - b = \dfrac{s - W_0 r}{C_\xi(R) - r}$

Thus $\quad b = -\dfrac{s - W_0 C_\xi(R)}{C_\xi(R) - r}$, *when* $b > 0$, *he will lend*; *when* $b < 0$, *he will borrow* (7)

The Determination of the Value of C_ξ (*R*)

The overall impression of stock market and exchange rate data is that normal distribution or similar symmetric distributions can be well fitted to the central data, yet there seem to be fat tails. Most importantly, there is empirical evidence that distribution of returns can possess fat or heavy tails and risk is spread unevenly so that a careful analysis of returns is required (Seal, 1969). In this context, we deal with the loss distribution and, especially, with parameters, which summarize the potential risk to some extent.

Extreme value theory's application in the fat tails of return distribution

Extreme value theory was discovered by Stephen Resnick (1987). A small group of theorists has recently developed this theory based upon the strong assumption of iid returns, which tells us that the limiting distribution of extreme returns has always the same form, whatever the unknown iid distribution from which the data are drawn.

Extreme value theory is critical for obtaining the exceedance values corresponding to given probability. So the estimation of lower tail index is dependable on the application of EV theory.

Modeling by Extreme Value Distribution:

The actual df of a maximum will be replaced by an extreme value (EV) df. Here is a list of three different submodels by writing down the standard EV dfs for the different shape parameters a:

$$Gumbel(EV0): G_0(x) = \exp(-e^{-x}), for\ all\ x;$$

$$Frechet(EV1), \alpha > 0: G_{1,\alpha}(x) = \exp(-x^{-\alpha}), x \geq 0; G_{1,\alpha}(x) = 0, x < 0;$$

$$Weibull(EV2), \alpha < 0: G_{2,\alpha}(x) = \exp(-(-x)^{-\alpha}), x \leq 0; G_{2,\alpha}(x) = 1, x > 0$$

EV densities are unimodal. Remember that a distribution and the pertaining density f are called unimodal if the density is non-decreasing left of some point u and non-increasing right of u. Then u is called a mode. Frechet densities are skewed to the right (Reiss, 2001).

Compared with normal distribution, the kurtosis β of a fat tail distribution is smaller. When =3, the distribution is normal. When β>3, the distribution has a sharp shape. When β<3, the distribution is fat. When β=1.8, the distribution is a line. When β<1.8, the distribution has U shape (Diao Mingbi, 1998).

One method of extracting lower extremes from a set of data $X_1, \ldots X_n$ is to take the exceedances lower a predetermined threshold u. Exceedances lower u (valleys-lower-threshold(pot)) are those X_i with X_i<u taken in the original order of their outcome or in any other order. The values (X_i-u) are the excesses lower u.

Subsequently, the number of exceedances lower u will be denoted by k or, occasionally, by K to emphasize the randomness of this number. In many cases, the values over u are not recorded or cannot be observed by the statistician. Given random variables $X_1, \ldots, X_n$, we may write

$$K = \sum_{i \leq n} I(X_i < u), i = 1, \ldots, n \tag{8}$$

Here $I(X_i < u)$ mean: $I(X_i < u) = 1, when\ X_i < u;\ otherwise I(X_i < u) = 0$

There is a greater variability in the estimates of the shape parameter for varying numbers k of extremes, which could be reduced, to some extent, by smoothing these estimates over a moving window. Omitting this extraordinary event from the data yields an underestimation of the risk entailed in negative returns.

The distribution features and parameter estimation of financial asset return

The empirical evidence in the stock market returns show that the returns are fat tailed. Koedijk (1990) and Jansen (2000) consider a limiting distribution G(x) of above three asymptotic distributions that is characterized by a lack of some higher moments, that is

$$Frechet(EV1), \alpha > 0: G_{1,\alpha}(x) = \exp(-x^{-\alpha}), x \geq 0; G_{1,\alpha}(x) = 0, x < 0; \tag{9}$$

Here, a is the lower tail index, which can be estimated using Hill's (1975) moment estimator,

$$\frac{1}{a} = \gamma = \frac{1}{k}\sum_{i=1}^{k} \ln(X_{n+1-i}) - \ln(X_{n-k}) \tag{10}$$

Therefore the lower tail index a is determined by k and the distribution of portfolio return. For stock market returns, one must take heavy upper or lower tails with a tail index around 3 into account (Reiss, 2001).

The opposite tail index of a distribution can be estimated by multiplying the data with (-1) and calculating order statistics from the tail.

One critical aspect of the Hill estimator is the choice of k. The number k of lower or upper extremes was chosen according to the insight gained from the diagrams of the estimators.

After calculating the lower tail index a, the quintiles η that will only be exceeded with probability p can be estimated by the following calculation using bootstrap procedure of Hall (1990):

$$\eta = X_{(n-k)}\left(\frac{k}{pn}\right)^{\frac{1}{a}} \tag{11}$$

Finally, C ξ $(R)$the and risk premium r_p can be calculated according to the following:

$$c = 1 - |\eta|$$

$$r_p = \frac{R - r}{r - C_\xi(R)} \tag{12}$$

Here, c is the value of , R is the gross rate of return of risk assets. r is the gross rate of return of risk-free asset. The probability of one, two or three occurrences of an event in a sample with size n is 1/n, 1/2n, 1/3n.

Therefore according to the Safety-First Portfolio Optimization Model, the investor will maximize the risk premium r_p to the potential maximum opportunity loss $(s - W_0 r)$ that he is willing to incur with probability ξ. Under this condition the investor will decide his optimal portfolio.

Sample and data

Based upon the historical data, under the premise of China insurance fund direct investment into China mainland capital market and Hong Kong capital market[4], we take it as our research object. Insurance fund trustees and managers are typically risk averse. However, the international investment of insurance fund can effectively diversify the home systematic risks, thus benefit from its investment into the global capital market. One feature of insurance fund international

[4] In fact, China State Council issued "Some Opinions from State Council on pushing the reform, open and stable development of China capital market" on the last day of January 2004, which allow QDII investment into overseas capital markets and CEPA with Hong Kong. Up to now, some relevant authorities of China have been drafting the detailed policies about China insurance fund investment into Hong Kong capital market. It is reported that some underground funds have already gone out of the door via "underground tubes" and invested into Hong Kong capital market.

investment is its "home assets bias" (E. Philip Davis, 2002). There are not only HANG SENG CHINA ENTERPRISES shares (H shares), but also HANG SENG CHINA AFFILIATED CORP shares (Red chips) in the capital market of Hong Kong. Furthermore, compared with those shares in Europe, US and Japan etc, HANG SENG blue chips and local shares etc. have more content of "home assets", which meet the "home bias" of insurance fund.[5] Thus, we put the whole capital market of Hong Kong into three parts: one is H shares, one is Red chips, one is Hang Seng index.[6]

The weekly stock market indices of SHANGHAI SE COMPOSITE (SH[7]), HANG SENG CHINA ENTERPRISES (HH), HANG SENG CHINA AFFILIATED CORP (HR) and HANG SENG (HK) are obtained from the world's largest financial DataStream. The weekly rate of return of SH, HH, HR and HK is denoted as SHR, HHR, HRR and HKR respectively.

The period of data that used in this research is from January 1st 2000 to November 23rd 2004 as the shorter period. As the stock market index of HH was not issued until July 20th 1993, we take the period from July 20th 1993 to November 23rd 2004 as the longer period. Assume the investor use these weekly stock indices to decide portfolio according to the S-F Model and M-V Model respectively, and the investor can borrow or lend money at the risk-free rate of return. Some descriptive return statistics such as mean, standard deviation, max, min, range, skewness, kurtosis and correlations for SHR, HHR, HRR and HKR are calculated and list in table 1 to 4.

Table 1 reports summary statistics for SHR, HHR, HRR and HKR. The weekly mean return over the studied shorter period for SHR is 0.0506%, for HHR 0.4574% (the biggest one, which means Hong Kong H shares' bullish trends during this shorter period), for HRR -0.0054922%, for HKR -0.0256%. So the weekly return among them differs sharply, and the weekly return for HHR is especially attracting. The weekly standard deviation for SHR, HHR, HRR and HKR is 2.9938%04.6668%04.7604%03.1002% respectively, without sharp discrepancy. The weekly minimum to maximum range for SHR is close to that for HKR. The weekly range for HKR is greatest, which means the fluctuating prices of Hong Kong Red Chips are widest. The weekly return skewness and kurtosis for SHR, HHR, HRR and HKR indicate that the weekly return for them all have fat tails, decisively rejecting normality, which is

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[5] More empirical researches have indicated that, the exchange risk between inland capital markets and Hong Kong capital market is very small, which meets the risk aversion attitudes of China insurance fund managers.

[6] These three parts may have overlapping or inclusive relations. But this kind of division may well suit for the "home assets bias" rule of insurance fund.

[7] For the convenience of this study, SHANGHAI SE COMPOSITE, HANG SENG CHINA ENTERPRISES, HANG SENG CHINA AFFILIATED CORP and HANG SENG is abbreviated as SH(ShangHai), HH(Hong kong H shares), HR(Hong kong Red chips) and HK(Hong Kong) respectively.

coincidence with our assumption that the limiting distribution of the tail behavior of the stock returns is fat tailed (this assumption has already been justified by many researchers' empirical studies).

Table 1: Descriptive return statistics for SHR, HHR, HRR and HKR (shorter period)

	SHR	HHR	HRR	HKR
Weekly mean	0.000506	**0.004574**	-0.000054922	-0.000256
Weekly S.D.	0.029938	0.046668	0.047604	0.031002
Max	0.119760	**0.232480**	0.229720	0.133710
Min	-0.101170	-0.126870	-0.180120	-0.106500
Range	0.22093	0.35935	0.40984	0.24021
Skewness	0.5886952	0.3093277	0.0204135	0.0570583
Kurtosis	2.3231559	1.9958686	2.2527229	1.1076847

Source: DataStream and the author's calculation. Weekly January 1, 2000 to November 23, 2004.

Table 2 reports the coefficient estimates for SHR, HHR, HRR and HKR during the studied shorter period. The coefficient between SHR and HHR is very close to that between SHR and HRR, and that between SHR and HKR. All the coefficient estimates are much lower, which indicate that the correlation between them is very weak. Coincidently, it is the low correlation between SHR and HHR (HRR, HKR) that is usually interpreted as an indication of the potential benefit for China insurance fund direct investment into Hong Kong capital market to optimize its international portfolio diversification.

Table 2: Correlations for SHR, HHR, HRR and HKR (shorter period)

	SHR	HHR	HRR	HKR
SHR	1.00000	**0.10851**	**0.13023**	**0.08170**
HHR	**0.10851**	1.00000	0.60850	0.44100
HRR	**0.13023**	0.60850	1.00000	0.79727
HKR	**0.08170**	0.44100	0.79727	1.00000

Source: DataStream and the author's calculation.

Note: The data is weekly and from January 1, 2000 to November 23, 2004.

Table 3 shows some descriptive statistics for SHR, HHR, HRR and HKR during the studied longer period. The weekly mean return for SHR, HHR, HRR and HKR is 0.2103%, 0.2536%, 0.1649% and 0.2075% respectively, very close to each other. Although the weekly mean return for HHR is still biggest among them during the longer period, it shrinks nearly 50% compared with that during the shorter period. But the weekly mean returns for SHR, HRR and HKR during the longer period increase sharply compared with that during the shorter period, which indicates a longer trend. The weekly standard deviation for SHR, HHR, HRR and

HKR is 5.0410%, 6.5125%, 6.3809% and 4.1399%respectively, also very close to each other. The weekly minimum to maximum range for SHR, HHR and HRR is nearly close to each other. The weekly range for HKR is lowest, indicating that the whole fluctuation in Hong Kong capital market is relatively narrow. The weekly minimum to maximum range for SHR is nearly 1.5 times of that for HKR, which is interpreted as a reason for China insurance fund direct investment into Hong Kong capital market. The weekly return skewness and kurtosis for SHR, HHR, HRR and HKR during the longer period indicate their fat tail distribution rather than normal distribution, which is coincidence with our assumption.

Table 3: Descriptive return statistics for SHR, HHR, HRR and HKR (longer period)

	SHR	HHR	HRR	HKR
Weekly mean	0.002103	0.002536	0.001649	0.002075
Weekly S.D.	0.050410	0.065125	0.063809	0.041399
Max	**0.436820**	0.394390	0.390250	0.190930
Min	-0.263910	-0.318690	-0.375750	-0.269550
Range	0.70073	0.71308	0.766	0.46048
Skewness	**1.8630805**	0.5815869	0.4069282	-0.1643712
Kurtosis	**15.3057431**	5.7149180	7.1944390	4.5941884

Source: DataStream and the author's calculation.

Note: The data is weekly and from July 20, 1993 to November 23, 2004.

Table 4 reports the coefficient estimates for SHR, HHR, HRR and HKR during the studied longer period. The coefficient estimates between SHR and HHR is very close to that between SHR and HRR, which is more than doubled that between SHR and HKR. But the entire coefficient estimates between SHR and HHR, between SHR and HRR, between SHR and HKR are much lower, which indicates the weak correlation between them. Therefore, from the view of longer period, China insurance fund direct investment into Hong Kong capital market can diversify its home systematic risks and maximize international portfolios.

Table 4: Correlations for SHR, HHR, HRR and HKR (longer period)

	SHR	HHR	HRR	HKR
SHR	**1.00000**	**0.15807**	**0.15500**	**0.06875**
HHR	**0.15807**	1.00000	0.80301	0.63051
HRR	**0.15500**	0.80301	1.00000	0.78217
HKR	**0.06875**	0.63051	0.78217	1.00000

Source: DataStream and the author's calculation.

Note: The data is weekly and from July 20, 1993 to November 23, 2004.

In this research we take the risk-free rate as 1.98% per year. Therefore the weekly risk-free rate is 0.038077%. The gross rate of return of risk-free asset is 1.00038077. In the next section we will use S-F Model to estimate the portfolio diversification results of China insurance fund investment into Hong Kong capital market.

The simulation results using S-F Model

First, as empirical evidence in the stock market returns show that distribution of returns can possess fat or heavy tails; hence portfolio management has to take this predictability into account. In technical terms, we may assume the stock returns are heavy or fat tailed. Then we assume the risk aversion safety-first China insurance fund manger use S-F Model to decide the optimal portfolio of his investment into Hong Kong capital market.

The simulated portfolio diversification results for the shorter and longer periods are calculated by the use of the S-V Model in Section one, the methodology in Section two and the sample and data in Section three, see table 5-10.

The portfolio diversification results for the shorter period

These results are obtained from 11 hypothetical portfolios of SH+HH, SH+HR, SH+HK, with the weights of SH varying from 100% to 0 by 10% step size, see the first and second column of these tables. The third column of these tables is the weekly mean return of these portfolios net of the risk-free rate. The fourth, fifth and sixth columns of these tables are the lower tail k, X_{n-k} and a respectively. The seventh and eighth columns of these tables are the lower tail exceedance level η for the two selected probabilities the investor is willing to incur his maximum acceptable opportunity loss[8]. The ninth, tenth, eleventh and twelfth columns of these tables calculate the risk premium under the two selected probabilities.

Using the S-F Model, the investor will choose the portfolio with the highest risk premium as his optimal portfolio among the 11 hypothetical portfolios. Table 5, 6, 7 show the results for the shorter period, while table 8, 9, 10 show the results for the longer period.

[8] Here the selected probabilities are the p1 and p2, which respectively represents the maximum acceptable opportunity loss will happen once or twice in the sample. In order to save space, we do not show the results for the case of three occurrences in the sample, and the results for the 95%CI. In general these results are basically the same, as the investor select the optimal portfolio with the highest risk premium according to the S-F rule.

Table 5: The optimal portfolio between SH and HH using S-F Model (shorter period)

Hypothetical Portfolios			Lower tail η			Lower tail		Risk premium r_p			
						p_1	p_2	p_1		p_2	
SH	HH	Weekly mean	k	X_{n-k}	a	η	η	$C_\zeta(R)$	r_p	$C_\zeta(R)$	r_p
1.00	0.00	0.00051	15	-0.03837	2.75512	-0.10253	-0.13186	0.89747	0.00126	0.86814	0.00098
0.90	0.10	0.00091	13	-0.03932	3.24873	-0.08660	-0.10719	0.91340	0.00608	0.89281	0.00492
0.80	0.20	0.00132	12	-0.04206	3.81199	-0.07726	-0.09267	0.92274	0.01210	0.90733	0.01009
0.70	0.30	0.00173	11	-0.04373	5.22483	-0.06920	-0.07902	0.93080	0.01939	0.92098	0.01699
0.60	0.40	0.00213	15	-0.03978	4.47011	-0.07291	-0.08514	0.92709	0.02387	0.91486	0.02045
0.50	0.50	0.00254	17	-0.03971	3.94885	-0.08138	-0.09699	0.91862	0.02641	0.90301	0.02218
0.40	0.60	0.00295	16	-0.04352	4.28289	-0.08314	-0.09775	0.91686	0.03076	0.90225	0.02618
0.30	0.70	0.00335	15	-0.04834	4.17125	-0.09252	-0.10925	0.90748	0.03196	0.89075	0.02708
0.20	0.80	0.00376	20	-0.04993	3.97469	-0.10609	-0.12631	0.89391	0.03174	0.87369	0.02667
0.10*	**0.90***	0.00417	17	-0.05954	4.35853	-0.11406	-0.13372	0.88594	**0.03311***	0.86628	**0.02826***
0.00	*1.00*	0.00457	17	-0.06540	4.08541	-0.13805	-0.15504	0.86195	0.03026	0.84496	0.02695

Source: DataStream and the author's calculation.

Note:
(1) The data is weekly and from January 1, 2000 to November 23, 2004.
(2) The risk-free rate is set as 1.98% per year. Therefore the weekly risk-free rate is 0.038077%. The gross rate of return of risk-free asset is 1.00038077.
(3) The gross rate of risk assets return is equal to one plus weekly mean net of the risk-free rate.
(4) The pre-specified critical probability value represents the maximal acceptable probability of a disaster. The sample size n of this shorter period is 256. So the probability of one occurrence in the sample is $p_1 = 1/256 = 0.00390625 = 0.390625\%$. The probability of two occurrence in the sample is $p_2 = 1/(2*256) = 0.0019535 = 0.19535\%$.
(5) The lower tail η shown in the table represents the lower tail exceedence levels or the expected number of occurrences for selected probabilities.
(6) *Indicates the optimal portfolio which has the highest risk premium among the available choices.

Table 6: The optimal portfolio between SH and HR using S-F Model (shorter period)

Hypothetical Portfolios			Lower tail η			Lower tail		Risk premium r_p			
								p_1		p_2	
SH	HH	Weekly mean	k	X_{n-k}	a	p_1 η	p_2 η	$C_\xi(R)$	r_p	$C_\xi(R)$	r_p
1.00*	**0.00***	0.00051	15	-0.03837	2.75512	-0.10253	-0.13186	0.89747	**0.00126***	0.86814	**0.00098***
0.90	0.10	0.00045	15	-0.03750	2.99987	-0.09249	-0.11653	0.90751	0.00075	0.88347	0.00059
0.80	0.20	0.00039	15	-0.03880	3.56264	-0.08314	-0.10100	0.91686	0.00011	0.89900	0.00009
0.70	0.30	0.00034	20	-0.03687	3.46099	-0.08762	-0.10704	0.91238	-0.00046	0.89296	-0.00038
0.60	0.40	0.00028	25	-0.03200	2.36687	-0.12467	-0.16709	0.87533	-0.00081	0.83291	-0.00060
0.50	0.50	0.00023	25	-0.03773	3.02029	-0.10953	-0.13779	0.89047	-0.00137	0.86221	-0.00109
0.40	0.60	0.00017	20	-0.04570	3.38525	-0.11072	-0.13588	0.88928	-0.00190	0.86412	-0.00155
0.30	0.70	0.00011	20	-0.04836	3.02332	-0.13026	-0.16383	0.86974	-0.00207	0.83617	-0.00165
0.20	0.80	0.00006	15	-0.05766	3.04656	-0.14025	-0.17609	0.85975	-0.00228	0.82391	-0.00182
0.10	0.90	0.0000011	20	-0.05815	3.08685	-0.15350	-0.19215	0.84650	-0.00247	0.80785	-0.00197
0.00	1.00	-0.00005	25	-0.05916	3.04592	-0.17078	-0.21443	0.82922	-0.00252	0.78557	-0.00201

Source: DataStream and the author's calculation.

Note: The data is weekly and from January 1, 2000 to November 23, 2004.

Table 7: The optimal portfolio between SH and HK using S-F Model (shorter period)

Hypothetical Portfolios			Lower tail η			Lower tail		Risk premium r_p			
								p_1		p_2	
SH	HH	Weekly mean	k	X_{n-k}	a	p_1 η	p_2 η	$C_\xi(R)$	r_p	$C_\xi(R)$	r_p
1.00*	**0.00***	0.00051	15	-0.03837	2.75512	-0.10253	-0.13186	0.89747	**0.00126***	0.86814	**0.00098***
0.90	0.10	0.00043	26	-0.03046	3.00925	-0.08994	-0.11323	0.91006	0.00055	0.88677	0.00043
0.80	0.20	0.00035	26	-0.02886	3.07427	-0.08271	-0.10362	0.91729	-0.00037	0.89638	-0.00030
0.70	0.30	0.00028	25	-0.02719	2.95121	-0.08093	-0.10235	0.91907	-0.00124	0.89765	-0.00098
0.60	0.40	0.00020	25	-0.03200	2.36687	-0.12467	-0.16709	0.87533	-0.00145	0.83291	-0.00108
0.50	0.50	0.00013	30	-0.03251	2.39022	-0.13419	-0.17915	0.86581	-0.00186	0.82085	-0.00140
0.40	0.60	0.00005	25	-0.03941	2.69634	-0.13004	-0.16816	0.86996	-0.00254	0.83184	-0.00196
0.30	0.70	-0.00003	30	-0.04018	2.64144	-0.14562	-0.18932	0.85438	-0.00281	0.81068	-0.00217
0.20	0.80	-0.00010	25	-0.05012	3.29604	-0.13415	-0.16555	0.86585	-0.00357	0.83445	-0.00290
0.10	0.90	-0.00018	25	-0.05592	3.40348	-0.14398	-0.17650	0.85602	-0.00388	0.82350	-0.00317
0.00	1.00	-0.00026	30	-0.05648	3.11844	-0.16810	-0.20994	0.83190	-0.00380	0.79006	-0.00305

Source: DataStream and the author's calculation.

Note: The data is weekly and from January 1, 2000 to November 23, 2004.

The optimal portfolios are those with highest risk premium. From table 5, 6, 7 for the shorter period, we find that the optimal portfolio of SH+HR and SH+HK consists of 100% SH, while the optimal portfolio of SH+HH consists of 10%SH and 90%HH, which indicates HH is the most attractive for the safety-first risk-aversion investor during the shorter period. Listed on Hong Kong capital market, Hong Kong H shares have the largest content of "home assets" which meets the preference of home assets bias for China insurance fund international investment (Philip, 2002). Therefore during the shorter period, the first choice of China insurance fund investment into Hong Kong capital market is Hong Kong H shares with the biggest content of home assets.

The portfolio diversification results for the longer period

From 8, 9, 10 for the longer period, we find that the optimal portfolio of SH+HH, SH+HR, SH+HK consists of 70%SH+30%HH, 80%SH+20%HR, 60%SH+40%HK respectively.

Table 8: The optimal portfolio between SH and HH using S-F Model (longer period)

Hypothetical Portfolios			Lower tail γ			Lower tail		Risk premium r_p			
								p_1		p_2	
SH	HH	Weekly mean	k	X_{n-k}	a	p_1 γ	p_2 γ	$C_\xi(R)$	r_p	$C_\xi(R)$	r_p
1.00	0.00	0.002103	30	-0.06633	3.16831	-0.19407	-0.24153	0.80593	0.00886	0.75847	0.00712
0.90	0.10	0.002146	30	-0.05922	2.82331	-0.19754	-0.25251	0.80246	0.00892	0.74749	0.00698
0.80	0.20	0.002190	42	-0.04267	2.82939	-0.18612	-0.23779	0.81388	0.00971	0.76221	0.00760
0.70*	**0.30***	**0.002233**	**42**	**-0.49730**	**2.94939**	**-0.17660**	**-0.22338**	**0.82340**	**0.01047***	**0.77662**	**0.00828***
0.60	*0.40*	*0.002276*	*66*	*-0.04215*	*2.74446*	*-0.19399*	*-0.24973*	*0.80601*	*0.00975*	*0.75027*	*0.00758*
0.50	0.50	0.002320	66	-0.04380	2.76422	-0.19940	-0.25623	0.80060	0.00971	0.74377	0.00756
0.40	0.60	0.002363	74	-0.04246	2.46872	-0.24274	-0.32143	0.75726	0.00816	0.67857	0.00616
0.30	0.70	0.002406	74	-0.04674	2.59404	-0.24563	-0.32876	0.75437	0.00824	0.67124	0.00616
0.20	0.80	0.002449	66	-0.05403	2.63430	-0.26506	-0.34487	0.73494	0.00779	0.65513	0.00599
0.10	0.90	0.002493	53	-0.06504	2.69272	-0.28414	-0.36755	0.71586	0.00743	0.63245	0.00574
0.00	1.00	0.002536	53	-0.07275	2.80872	-0.29904	-0.38275	0.70096	0.00720	0.61725	0.00563

Source: DataStream and the author's calculation.

Note: (1) The data is weekly and from July 20, 1973 to November 23, 2004.

(2) For the sake of conservative, the risk-free rate is set as 1.98% per year. Therefore the weekly risk-free rate is 0.038077%. The gross rate of return of risk-free asset is 1.00038077.

(3) The gross rate of risk assets return is equal to one plus weekly mean net of the risk-free rate.

(4) The pre-specified critical probability value represents the maximal acceptable probability of a disaster. The sample size n of this shorter period is 592. So the

probability of one occurrence in the sample is =1/592=0.001689=0.16895%. The probability of two occurrence in the sample is =1/(2*592)=0.000845=0.0845%.

(5) The lower tail shown in the table represents the lower tail exceedence levels or the expected number of occurrences for selected probabilities.

(6) *Indicates the optimal portfolio which has the highest risk premium among the available choices.

Table 9: The optimal portfolio between SH and HR using S-F Model (longer period)

Hypothetical Portfolios			Lower tail η			Lower tail		Risk premium r_p			
SH	HH	Weekly mean	k	X_{n-k}	a	p_1 η	p_2 η	p_1 $C_\zeta(R)$	p_1 r_p	p_2 $C_\zeta(R)$	p_2 r_p
1.00	0.00	0.002103	30	-0.06633	3.16831	-0.19407	-0.24153	0.80593	0.00886	0.75847	0.00712
0.90	0.10	0.002058	25	-0.06657	3.24673	-0.17941	-0.22211	0.82059	0.00933	0.77789	0.00754
0.80*	**0.20***	0.002012	25	-0.06399	3.47228	-0.16170	-0.19743	0.83830	**0.01007***	0.80257	**0.00825***
0.70	*0.30*	0.001967	42	-0.05010	2.97675	-0.17585	-0.22196	0.82415	0.00901	0.77804	0.00714
0.60	0.40	0.001921	42	-0.05084	2.92825	-0.18222	-0.23088	0.81778	0.00844	0.76912	0.00666
0.50	0.50	0.001876	53	-0.04802	2.73092	-0.20550	-0.26488	0.79450	0.00727	0.73512	0.00564
0.40	0.60	0.001831	66	-0.04718	2.72564	-0.21944	-0.28299	0.78056	0.00660	0.71701	0.00512
0.30	0.70	0.001785	53	-0.05496	2.70295	-0.23877	-0.30856	0.76123	0.00587	0.69144	0.00455
0.20	0.80	0.001740	74	-0.05051	2.50260	-0.28203	-0.37203	0.71797	0.00482	0.62797	0.00365
0.10	0.90	0.001694	66	-0.05614	2.33191	-0.33850	-0.45566	0.66150	0.00388	0.54434	0.00288
0.00	1.00	0.001649	53	-0.06828	2.35728	-0.36793	-0.49370	0.63207	0.00345	0.50630	0.00257

Source: DataStream and the author's calculation.

Note: The data is weekly and from July 20, 1973 to November 23, 2004.

Table 10: The optimal portfolio between SH and HK using S-F Model (longer period)

Hypothetical Portfolios			Lower tail γ			Lower tail		Risk premium r_p			
SH	HH	Weekly mean	k	X_{n-k}	a	p_1 γ	p_2 γ	p_1 $C_\xi(R)$	p_1 r_p	p_2 $C_\xi(R)$	p_2 r_p
1.00	0.00	0.002103	30	-0.06633	3.16831	-0.19407	-0.24153	0.80593	0.00886	0.75847	0.00712
0.90	0.10	0.002100	30	-0.05850	2.88376	-0.19027	-0.24197	0.80973	0.00902	0.75803	0.00710
0.80	0.20	0.002097	44	-0.04536	2.68266	-0.18591	-0.24072	0.81409	0.00922	0.75928	0.00712
0.70	0.30	0.002095	42	-0.04393	2.87639	-0.16110	-0.20500	0.83890	0.01062	0.79500	0.00835
0.60*	**0.40***	0.002092	53	-0.03602	2.72396	-0.15472	-0.19955	0.84528	**0.01104***	0.80045	**0.00856***
0.50	0.50	0.002089	66	-0.03263	2.63146	-0.16035	-0.20868	0.83965	0.01063	0.79132	0.00817
0.40	0.60	0.002086	74	-0.03151	2.63423	-0.16145	-0.21005	0.83855	0.01054	0.78995	0.00811
0.30	0.70	0.002083	74	-0.03151	2.42178	-0.18633	-0.24808	0.81367	0.00912	0.75192	0.00685
0.20	0.80	0.002081	81	-0.03285	2.52531	-0.18719	-0.24631	0.81281	0.00907	0.75369	0.00690
0.10	0.90	0.002078	66	-0.03870	2.50052	-0.20672	-0.27275	0.79328	0.00820	0.72725	0.00622
0.00	1.00	0.002075	66	-0.04074	2.27414	-0.25711	-0.34873	0.74289	0.00658	0.65127	0.00486

Source: DataStream and the author's calculation.
Note: The data is weekly and from July 20, 1973 to November 23, 2004.

These simulating results using S-F Model indicate that, compared with the shorter period, China insurance fund manager will gradually increase the investment proportion of Hong Kong Red chips and Hong Kong Blue chips or local shares except Hong Kong H shares during the longer period[9]. Supposedly, from the point of longer investment, China insurance fund investment into Hong Kong capital market can not only effectively diversify its home systematic risks[10], but also can make use of overseas capital market to maximize its portfolio so as to increase its return of investment[11]. To sum up the above simulating results, see the following table:

[9] In fact, from the view point of whole Hong Kong capital market, the contents of home assets for Hong Kong H shares (HH) are larger than that for Hong Kong Red Chips (HR), and the contents of home assets for Hong Kong Red chips are larger than that for Hong Kong Blue Chips or local shares (HK), which implies the considering sequence of China insurance fund investment into Hong Kong capital market for a specified term.

[10] More empirical researches from both academic and practitioners have found that the market fluctuating risks of emerging market are far greater than that of mature capital markets.

[11] In fact, from the view point of whole global capital markets, the contents of home assets in Hong Kong capital market are larger than that in New York, London, Tokyo or Singapore capital markets, which implies the considering sequence for safety-first risk-aversion manager of China insurance fund to invest in global capital markets.

Therefore, the successful experiences of China insurance fund investment into Hong Kong capital market will demonstrate great significance for China insurance fund investment further in overseas capital markets such as New York, London, Tokyo, Singapore even PRChina Taiwan capital markets: on the one hand to diversify its home systematic risks; on the other hand to optimize its portfolios effectively by the use of overseas capital markets, so as to increase its return of investment.

Table 11: Summary of the optimal portfolios among SH and HH, HR, HK using S-F Model

	SH+HH Portfolio		SH+HR Portfolio		SH+HK Portfolio	
	SH	HH	SH	HR	SH	HK
The shorter period	0.10	0.90	1.00	0.00	1.00	0.00
The longer period	0.70	0.30	0.80	0.20	0.60	0.40

The application of S-F Model in a manager's investment decision

In this subsection we will apply the above results to illustrate how a safety-first risk-averse manager to make investment decision according to S-F rule with an optimal 70%SH and 30%HH portfolio in the longer period as in table 8.

Table 8 shows that the net weekly mean return for this optimal portfolio is 0.002233 (or 0.2233%). Therefore the gross rate of return for risk assets is 1.002233 (or 100.2233%). As pre-specified, the weekly net risk-free rate of return is 0.00038077 (or 0.038077%). Thus the gross rate of return for risk-free asset is 1.00038077(or 100.038077%).

Now we imagine that a insurance fund manager with typical safety-first risk-aversion attitude has set his critical wealth s to equal to 80% of his total initial wealth W, that is

$$s = 80\%W$$

Therefore, the manager's minimally acceptable gross return is 80%, or in other words, the maximal acceptable net negative return is –20%.

We assume that the manager's maximal acceptable probability of this worst outcome is equal to one chance in this sample size, that is

$$p = \frac{1}{592} = 0.001689 = 0.16895\%$$

Table 8 also shows the exceedance value for this optimal portfolio with the maximum acceptable probability of 0.16895%, that is

$$C_\xi(R) = 0.82340$$

(i) The manager's profit analysis

We assume that the manager's total initial wealth W is in the form of equity, and he will borrow or lend in the financial market based upon the S-F Model. Then his lending or borrowing volume b is decided upon the following:

$$b = -\frac{s - W_0 C_\xi(R)}{C_\xi(R) - r}\text{ , when } b > 0, \text{he will lend; when } b < 0 \text{ , he will borrow}$$

Thus $b = -\frac{0.8W - W*0.82340}{0.82340 - 1.00038077} = -0.132218W$, which means that the manager should leverage or borrow 0.132218W in the financial market in order to maximize his return of optimal portfolio.

The manager's total investment including normalized wealth and leverage is:

W + 0.132218W=1.132218W

The gross return from his investment is:

1.132218W*(1+0.002233[12] + 0.00038077)=1.135177W

His financing cost is:

0.132218W*(1+0.00038077)=0.132268W

Thus, the net return from his investment (including his initial wealth) is:

1.135177W-0.132268W=1.002909W[13]

(ii) The manager's maximum loss analysis

If the disaster for the manager with maximum acceptable probability actually occurs (that is, the maximum acceptable opportunity loss occurs once in the size of the sample), then the manager's portfolio's return is limited at most 20% loss based on the safety-first rule.

As the exceedence value of the manager's maximum acceptable opportunity loss is 0.82340,

The gross return from his investment is:

1.132218W*0.82340=0.932268W

His financing cost is:

0.132218W*(1+0.00038077)=0.132268W

Thus, the net return from his investment (including his initial wealth) is:

0.932268W-0.132268W=0.8W

That is to say, under the condition that the maximum acceptable opportunity loss actually happens, the wealth of the manager shrinks to 80% of his initial wealth, which is coincident with his pre-set critical wealth.

(iii) The comparative analysis between optimal portfolio and sub-optimal portfolio

The optimal portfolio in table 8 is 70%SH+30%HH0

Now we consider a sub-optimal portfolio in table 8 such as 60%SH+40%HH0The weekly net mean return of this portfolio is 0.002276. The exceedence value with maximum acceptable opportunity loss probability is $C_{\xi}(R) = 0.80601$, then

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[12] The net weekly mean return in the third column of table 8 is based upon the net rate of risk-free asset.

[13] We notice that the return here is specified to weekly period, so it will look bigger when specified to monthly or yearly period.

$$b = \frac{s - WC_{\xi}(R)}{C_{\xi}(R) - r} = \frac{0.8W - 0.80601W}{0.80601 - 1.00038077} = -0.030920W$$

Under this sub-optimal portfolio, the manager will borrow 0.030920W by leverage in the financial market in order to maximize his portfolio.

The manager's total investment including normalized wealth and leverage is:

W+0.030920W=1.030920W

The gross return from his investment is:

1.030920W *(1+0.002276+0.00038077)=1.033658W

His financing cost is:

0.030920W*(1+0.00038077)=0.030932W

Thus, the net return from his investment (including his initial wealth) is:

1.033658W-0.030932W=1.002727W<1.002909W

If the disaster for the manager with the maximum acceptable opportunity loss actually occurs, then the manager's portfolio's return is limited at most 20% loss based on the safety-first rule.

Table 8 shows that the exceedance value of the manager's maximum acceptable opportunity loss is 0.80601, then

The gross return from his investment is:

1.030920W *0.80601=0.830932W

His financing cost is:

0.030920W *(1+0.00038077)=0.030932W

Thus, the net return from his investment (including his initial wealth) is:

0.830932W -0.030932W =0.8W

That is to say, under the condition that the maximum acceptable opportunity loss actually happens, the wealth of the manager shrinks to 80% of his initial wealth, which is coincident with his pre-set critical wealth.

By comparing the above results under the conditions of both optimal portfolio and the sub-optimal portfolio, we find the optimal portfolio brings net return of 1.002909W, which is bigger than 1.002727W that the sub-optimal brings, although based on the same critical wealth.

The Simulation Results Using M-V Model

In the year of 1952, as a Ph. D. candidate from department of economics in Chicago University, based upon the probability theory and quadratic programming method, Markowitz first used expected return and variance (or standard deviation) of risk assets to study the assets selection and portfolio. Then his paper that was published Journal of Finance signaled the starting point of Modern Asset Portfolio Theory. In the year of 1959, he systematically summarized his asset portfolio theory, which is the famous and traditional Mean-Variance Portfolio Optimization Model.

According to his theory, we assume that there are two kinds of securities A and B. The expected rate of return for A and B is r_1 and r_2 respectively. The standard variance for A and B is σ_1 and σ_2 respectively. The effective portfolio is $X_p = \{x_1, x_2\}$, $x_1 + x_2 = 1, x_1 \geq 0, x_2 \geq 0$.

Then the expected rate of return and variance of this portfolio can be obtained according the following:

$$r_p = x_1 r_1 + x_2 r_2 = x_1 r_1 + (1 - x_1) r_2$$

$$\sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2 x_1 x_2 \rho \sigma_1 \sigma_2$$

Here, ρ is the correlation coefficient between r_1 and r_2, $|\rho| \leq 1$

Under the traditional M-V Model, the investor will maximize the expected rate of return while minimize the variance of the portfolio. Therefore he will select the optimal portfolio with the highest Sharpe ratio or lowest CV.

Now based on the data in Section three, we use the above traditional M-V Model and its methodology to select the optimal portfolios of China insurance fund investment into Hong Kong capital market for the shorter period and the longer period, see table 11-18.

The portfolio diversification results for the shorter period

The portfolio diversification results for the shorter (or longer) period are obtained from 11 hypothetical portfolios of SH+HH, SH+HR, SH+HK, with the weights of SH varying from 100% to 0 by 10% step size, see the first and second column of these tables. The third column of these tables is the weekly mean return of these portfolios net of the risk-free rate. The fourth and fifth columns of these tables is the variance and standard deviation of the portfolio respectively. The sixth and seventh columns of these tables are Sharpe ratio and Coefficient of Variation respectively.

Using the traditional M-V Model, the investor will choose the portfolio with the highest SP or least CV as his optimal portfolio among the 11 hypothetical portfolios. Table 12, 13, 14 show the results for the shorter period, while table 15, 16, 17 show the results for the longer period.

Table 12: The optimal portfolio between SH and HH using M-V Model (shorter period)

Hypothetical Portfolios		M	V	SD	SP	CV
SH	HH					
1.00	0.00	0.00051	0.00090	0.02994	0.00418	59.16601
0.90	0.10	0.00091	0.00078	0.02784	0.01911	30.49941
0.80	0.20	0.00132	0.00071	0.02663	0.03525	20.18170
0.70	0.30	0.00173	0.00070	0.02644	0.05090	15.31281
0.60	0.40	0.00213	0.00074	0.02727	0.06425	12.78571
0.50	0.50	0.00254	0.00084	0.02906	0.07431	11.44003
0.40	0.60	0.00295	0.00100	0.03163	0.08114	10.73241
0.30	0.70	0.00335	0.00121	0.03481	0.08541	10.37893
0.20	0.80	0.00376	0.00148	0.03845	0.08790	10.22435
0.10	0.90	0.00417	0.00180	0.04243	0.08924	10.18203
0.00*	**1.00***	0.00457	0.00218	0.04667	**0.08985***	**10.20289***

Source: DataStream and the author's calculation.

Note:
(1) The data is weekly and from January 1, 2000 to November 23, 2004.
(2) M, V, SD, SP, CV shown in the table represents weekly return mean, the variance, the standard deviation, Sharpe Ratio[14] and Coefficient of Variation respectively.
(3) The risk-free rate is set as 1.98% per year. Therefore the weekly risk-free rate is 0.038077%. The gross rate of return of risk-free asset is 1.00038077.
(4) *Indicates the optimal portfolio which has the highest SP ratio or least CV ratio among the available choices.

Table 13: The optimal portfolio between SH and HR using M-V Model (shorter period)

Hypothetical Portfolios		M	V	SD	SP	CV
SH	HH					
1.00*	**0.00***	0.00051	0.00090	0.02994	**0.00418***	**59.16601***
0.90	0.10	0.00045	0.00078	0.02797	0.00247	62.15790
0.80	0.20	0.00039	0.00072	0.02690	0.00048	68.30840
0.70	0.30	0.00034	0.00072	0.02685	-0.00160	79.51181
0.60	0.40	0.00028	0.00077	0.02783	-0.00356	98.80593
0.50	0.50	0.00023	0.00088	0.02972	-0.00522	131.78274
0.40	0.60	0.00017	0.00105	0.03238	-0.00653	191.07780
0.30	0.70	0.00011	0.00127	0.03562	-0.00751	314.26512
0.20	0.80	0.00006	0.00155	0.03931	-0.00823	686.55490
0.10	0.90	0.0000011	0.00188	0.04334	-0.00876	37032.35429
0.00	1.00	-0.00005	0.00227	0.04760	-0.00915	-866.75649

Source: DataStream and the author's calculation.

Note: The data is weekly and from January 1, 2000 to November 23, 2004.

[14] $SP = \frac{R_t - r_f}{\sigma}, CV = \frac{\sigma}{M}$

Table 14: The optimal portfolio between SH and HK using M-V Model (shorter period)

Hypothetical Portfolios		M	V	SD	SP	CV
SH	HH					
1.00*	**0.00***	0.00051	0.00090	0.02994	**0.00418***	**59.16601***
0.90	0.10	0.00043	0.00075	0.02737	0.00179	63.68645
0.80	0.20	0.00035	0.00064	0.02523	-0.00108	71.33942
0.70	0.30	0.00028	0.00056	0.02361	-0.00438	85.11910
0.60	0.40	0.00020	0.00051	0.02265	-0.00793	112.55451
0.50	0.50	0.00013	0.00050	0.02241	-0.01141	179.29040
0.40	0.60	0.00005	0.00053	0.02293	-0.01448	469.88733
0.30	0.70	-0.00003	0.00058	0.02416	-0.01690	-881.56965
0.20	0.80	-0.00010	0.00068	0.02599	-0.01864	-250.82338
0.10	0.90	-0.00018	0.00080	0.02830	-0.01981	-157.41995
0.00	1.00	-0.00026	0.00096	0.03100	-0.02054	-121.10156

Source: DataStream and the author's calculation.

Note: The data is weekly and from January 1, 2000 to November 23, 2004.

The optimal portfolios are those with highest SP or least CV. From table 11, 12, 13 for the shorter period, we find that the optimal portfolio of SH+HR and SH+HK consists of 100% SH, while the optimal portfolio of SH+HH consists of 100%HH, which indicates HH is the most attractive during the shorter period.

The portfolio diversification results for the longer period

Table 15: The optimal portfolio between SH and HH using M-V Model (longer period)

Hypothetical Portfolios		M	V	SD	SP	CV
SH	HH					
1.00	0.00	0.002103	0.002541	0.050410	0.034164	23.970518
0.90	0.10	0.002146	0.002194	0.046842	0.037691	21.824507
0.80	0.20	0.002190	0.001962	0.044295	0.040836	20.229778
0.70	0.30	0.002233	0.001845	0.042952	0.043121	19.235789
0.60*	**0.40***	0.002276	0.001843	0.042924	**0.044157***	**18.857963***
0.50	0.50	0.002320	0.001955	0.044216	0.043847	19.062835
0.40	0.60	0.002363	0.002183	0.046718	0.042426	19.772124
0.30	0.70	0.002406	0.002525	0.050248	0.040307	20.883658
0.20	0.80	0.002449	0.002982	0.054609	0.037881	22.294748
0.10	0.90	0.002493	0.003554	0.059617	0.035425	23.916834
0.00	1.00	0.002536	0.004241	0.065125	0.033094	25.680205

Source: DataStream and the author's calculation.

Note: (1) The data is weekly and from July 20, 1993 to November 23, 2004.
(2) M, V, SD, SP, CV shown in the table represents weekly return mean, the variance, the standard deviation, Sharpe Ratio and Coefficient of Variation respectively.
(3) For the sake of conservative, the risk-free rate is set as 1.98% per year. Therefore the weekly risk-free rate is 0.038077%. The gross rate of return of risk-free asset is 1.00038077.
(4) *Indicates the optimal portfolio which has the highest SP ratio or least CV ratio among the available choices.

Table 16: The optimal portfolio between SH and HR using M-V Model (longer period)

Hypothetical Portfolios		M	V	SD	SP	CV
SH	HH					
1.00	0.00	0.002103	0.002541	0.050410	0.034164	23.970518
0.90	0.10	0.002058	0.002189	0.046785	0.035841	22.737496
0.80	0.20	0.002012	0.001949	0.044145	0.036956	21.938528
0.70*	**0.30***	0.001967	0.001821	0.042673	**0.037167***	**21.696854***
0.60	0.40	0.001921	0.001806	0.042492	0.036257	22.115250
0.50	0.50	0.001876	0.001902	0.043617	0.034281	23.250207
0.40	0.60	0.001831	0.002112	0.045953	0.031550	25.102677
0.30	0.70	0.001785	0.002433	0.049327	0.028472	27.631259
0.20	0.80	0.001740	0.002867	0.053544	0.025381	30.776205
0.10	0.90	0.001694	0.003413	0.058422	0.022485	34.479521
0.00	1.00	0.001649	0.004072	0.063809	0.019875	38.695573

Source: DataStream and the author's calculation.

Note: The data is weekly and from July 20, 1993 to November 23, 2004.

Table 17: The optimal portfolio between SH and HK using M-V Model (longer period)

Hypothetical Portfolios		M	V	SD	SP	CV
SH	HH					
1.00	0.00	0.002103	0.002541	0.050410	0.034164	23.970518
0.90	0.10	0.002100	0.002101	0.045840	0.037509	21.826519
0.80	0.20	0.002097	0.001741	0.041723	0.041143	19.892760
0.70	0.30	0.002095	0.001460	0.038206	0.044858	18.240129
0.60	0.40	0.002092	0.001258	0.035467	0.048243	16.955262
0.50	0.50	0.002089	0.001135	0.033697	0.050694	16.130766
0.40*	**0.60***	0.002086	0.001092	0.033052	**0.051598***	**15.843279***
0.30	0.70	0.002083	0.001129	0.033597	0.050678	16.126090
0.20	0.80	0.002081	0.001244	0.035277	0.048186	16.955026
0.10	0.90	0.002078	0.001439	0.037940	0.044729	18.259915
0.00	1.00	0.002075	0.001714	0.041399	0.040924	19.951325

Source: DataStream and the author's calculation.
Note: The data is weekly and from July 20, 1993 to November 23, 2004.

From 13, 16, and 17 for the longer period, we find that the optimal portfolio of SH+HH, SH+HR, SH+HK consists of 60%SH+40%HH, 70%SH+30%HR, 40%SH+60%HK respectively, among which the overseas investment proportions are higher than that results using S-F Model.

To sum up the above simulating results of table 12 to 17, see the following table:

Table 18: Summary of the optimal portfolios among SH and HH, HR, HK using M-V Model (1)

	SH+HH Portfolio		SH+HR Portfolio		SH+HK Portfolio	
	SH	HH	SH	HR	SH	HK
The shorter period	0.00	1.00	1.00	0.00	1.00	0.00
The longer period	0.60	0.40	0.70	0.30	0.40	0.60

Table 19: Summary of the optimal portfolios among SH and HH, HR, HK using M-V Model (2)

	Weekly M	Weekly V	Highest SP
The shorter period			
0.00SH+1.00HH**	0.00457	0.00218	**0.08985****
1.00SH+0.00HR	0.00051	0.02994	0.00418
1.00SH+0.00HK	0.00051	0.00090	0.00418
The longer period			
0.60SH+0.40HH	0.002276	0.001843	0.044157
0.70SH+0.30HR	0.001967	0.001821	0.037167
0.40SH+0.60HK*	0.002086	0.001092	**0.051598***

From table 18 and 19, by comparing the highest SP, we find the tangential Sharpe ratio is higher in the shorter period, and that the most optimal portfolio in the shorter period consists of 100%HH. This result seems to indicate that Hong Kong H shares are extremely attractive in the shorter period.[15]

Furthermore, the tangential optimal portfolio for the longer period consists of 40%SH+60%HK, which indicates that from the point of longer period the insurance fund manager will gradually increase the overseas investment proportion so as to diversify home systematic risks and optimize portfolio.

[15] It is reported that in China some hot money flew by "underground channel" into Hong Kong capital market before China authorities allow home capital investment into Hong Kong capital market. The great price differences between A shares in home capital markets and H shares in Hong Kong capital market for the same listed company may be the prime reason for the driving of H shares.

Taking the highest SP of both the shorter and longer periods together, we find that the optimal portfolio with 100%HH has the highest SP, which indicates that Hong Kong H shares are especially attractive both in the shorter period and the longer period.

Comparison Between the Two Simulating Results

The estimated diversification results using M-V Model are contrasted with that using S-F Model as the following:

Table 19: Comparison of the optimal portfolios using S-F Model and M-V Model

	SH+HH Portfolio		SH+HR Portfolio		SH+HK Portfolio	
	SH	HH	SH	HR	SH	HK
The shorter period						
S-F Model	0.10	0.90	*1.00*	*0.00*	*1.00*	*0.00*
M-V Model	0.00	**1.00**	*1.00*	*0.00*	*1.00*	*0.00*
The longer period						
S-F Model	0.70	0.30	0.80	0.20	0.60	0.40
M-V Model	0.60	**0.40**	0.70	**0.30**	0.40	**0.60**

From the above table and the contrast, we find that:

(1) In the same shorter period for the portfolios such as SH+HR, SH+HK, the optimal portfolio using both models consists of 100%SH. While for the portfolios of SH+HH, the optimal portfolio using S-F Model consists of 90%HH, while the optimal portfolio using M-V Model consists of 100%HH, which indicates that Hong Kong H shares are most attractive in the shorter period.

(2) For all the portfolios both in the shorter period and the longer period, we astonishingly find that the overseas investment proportions (such as HH, HR and HK) using S-F Model are less than or at least no more than that using M-V model.

Does this surprising finding indicate that according to some unknown common reference the risk attitude using M-V Model is more active than the risk attitude using S-F Model, or the risk attitude using S-F Model is more conservative than the risk attitude using M-V Model?[16] If so, then does a rule exist with more broad premises that extend and include that of both models? If so, then what characteristics does this rule have? "Dialectic is a theory which maintains that something-more especially, human thought-develops in a way characterized by what is called the dialectic triad: thesis, antithesis, and synthesis. First there is some idea or

[16] Here we notice that this kind of conjecture is only limited to the two Models, that is S-F Model and M-V Model.

theory or movement which may be called a 'thesis'……The opposing idea or movement is called the 'antithesis'……by recognizing their respective values and by trying to preserve the merits and to avoid the limitations of both. This solution, which is the third step, called the 'syntheses'. Once attained, the syntheses in its turn may become the first step of a new dialectic triad, and it will do so if the particular synthesis reached turns out to be one-sided or otherwise unsatisfactory. The dialectic triad will thus proceed on a higher level, and it may reach a third level when a second synthesis has been attained." (Routledge & K. Paul, 1989).

Conclusion

In this paper we have summarized in detail the S-F Model that was first discovered by Roy in 1952 and then developed by followers such as Arzac, Bawa, Jansen, Mahfuzul Haque etc. S-F rule was highly praised by Markowitz in 1999 along with his famous paper published in 1952 that won him the Nobel Laureate Prize.

Under the premise that China insurance fund may be allowed to invest in Hong Kong capital markets, we used both M-V Model and S-F Model to the predicting analysis on the China insurance fund investment into mainland China capital market and Hong Kong capital market respectively. We have astonishingly found that: (1) In the same short period for the portfolios such as SH+HR, SH+HK, the optimal portfolio using both models consists of 100%SH. While for the portfolios of SH+HH, the optimal portfolio using S-F Model consists of 90%HH, but the optimal portfolio using M-V Model consists of 100%HH, which indicates that Hong Kong H shares are most attractive in the short period. (2) For all the portfolios both in the short period and the long period, we astonishingly found that the overseas investment proportions (such as HH, HR and HK) using S-F Model are less than or at least no more than that using M-V model.

This surprising finding indicates that according to some common references the risk attitude using M-V Model is more active than the risk attitude using S-F Model, or the risk attitude using S-F Model is more conservative than the risk attitude using M-V Model.

However, with the rapidly development of today's capital market, the assumption of Markowitz's Mean-Variance Portfolio Optimization Model[17] is gradually becoming less suitable to the reality of today's capital market. In fact, with the rapid development of modern capital market, the maximum return of an investor may well be far above the mean level, and the maximum loss of an investor may well be far below the mean level, or even near –100%. Then the real return of portfolio is not subject to normal distribution. More generally, the real return exists some

[17] Assumption one: the uncertain return of portifolios is subject to normal distribution. Assumption two: the utility function of investors over return of portfolios is quadratic.

skewness to mean level. In fact, with the increase of wealth, many investors have increased their investment proportion of the higher risk assets, i.e. their risk bearing is increasing with their wealth. In a word, the assumptions implied in M-V Model are facing a substantial crisis. Meanwhile, today's fast developing global capital markets have made the risks and uncertainties more and more complex, especially the possibility of the portfolio loss associated with infrequent catastrophic events makes us pay more attention to Roy's Safety-First rule. However, Roy's S-F Model itself needs improving so as to adapt to the market reality and investment uncertainty. Roy's S-F Model has implied these assumptions: (1) A dreadful event might substantially erode an investor's wealth, making safety-first germane to questions about portfolio optimization. (2) The equity distributions are subject to extreme returns or fat/ heavy tail. (3) The investor will pre-specify his critical wealth level with the maximum acceptable probability of a disaster. However, Roy's rule itself was based upon the mean and variance of the portfolio return. That is, in his S-F rule, Roy used only simple variance method to measure the risk of the portfolio. Furthermore, S-F Model gives the expected wealth maximization under constraints and pre-specified critical wealth level and probability for the investor.

By comparing the above M-V Model and S-F Model, we may find that M-V Model is biased towards the study of the mean and variance of portfolio return, while S-F model is biased to the study of extreme value of portfolio return (especially the lower tail); M-V Model is biased to study the risk of the portfolio itself (that is maximization of Sharpe ratio), while S-F Model is biased to study the risk bearing of the investor himself (that is maximization of his expected wealth or risk premium).

Therefore, with the fast development of modern capital markets, all uncertainties and risks are becoming more and more intricacate. Investors should highly value both the study of risks of assets and the study of risk bearing of investors, and should pay greater attention to both the objective risk of portfolio itself and the subjective risk bearing of the investor himself.

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